

A Clustering Algorithm for Block-Cave Production Scheduling

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Abstract: Production scheduling is one of the most important steps in the block-caving design process. Optimum production scheduling could add significant value to a mining project. The goal of long-term mine production scheduling is to determine the mining sequence, which optimizes the company's strategic objectives while honouring the operational limitations over the mine life. Mathematical programming with exact solution methods is considered a practical tool to model block-caving production scheduling problems; this tool makes it possible to search for the optimum values while considering all of the constraints involved in the operation. This kind of model seeks to account for real-world conditions and must respond to all practical problems which extraction procedures face. Consequently, the number of subjected constraints is considerable and has tighter boundaries, solving the model is not possible or requires a lot of time. It is thus crucial to reduce the size of the problem meaningfully by using techniques which ensure that the absolute solution has less deviation from the original model. This paper presents a clustering algorithm to reduce the size of the large-scale models in order to solve the problem in a reasonable time. The results show a significant reduction in the size of the model and CPU time. Application and comparison of the production schedule based on the draw control system with the clustering technique is presented using 2,487 drawpoints to be extracted over 32 years.

Keywords: Clustering, Production scheduling, Block caving, Draw control system.

1. INTRODUCTION

Block and panel caving have become the underground bulk mining methods of choice and are expected to continue in the foreseeable future [1]. Block caving is a complex and large-scale mining method. The application of block caving is for low-grade, caveable, and massive ore-bodies. Block cave mines demand a large capital investment for the development and construction of any production units. Planning of block caving operations poses complexities in different areas of mining including production scheduling. Production scheduling consists of defining the source, destination and extraction time of ore and waste during the life of mine. Strategic planning of any mining system has an enormous effect on the economics of the operation. Production scheduling is one of the key components in determining mine viability because the mining industry faces lower grade and marginal reserves. In block caving projects, deviations from optimal mine plans may result in significant financial losses, future financial liabilities, resource sterilization, and unbalanced cave subsidence, fragmentation size distribution and flow of muck resulting in potential infrastructure instability.

Optimization of long-term production scheduling is a significant aspect of mine planning. Determining the period and sequence of drawing and displacement of ore and waste is the aim of mine planning. Such scheduling must maximize the overall discounted net revenue from the mine within the existing economic, technical and environmental constraints [2].

Block-caving scheduling has been the subject of a lot of research. Most studies have applied mathematical programming, simulation and stochastic approaches. A mathematical model should not over- or under-estimate the value of the operation and has to solve models in a reasonable CPU time for a large-scale block-caving operation. Such a model, which seeks to account for real-world conditions, must respond to all practical problems which extraction procedures face. This means that numerous constraints must be built into the model; consequently, the size of the model increases substantially. If the number of subjected constraints is considerable and has tighter boundaries, solving the model is not possible or requires a lot of time. It is thus crucial to reduce the size of the problem meaningfully by using techniques which ensure that the absolute solution has less deviation from the original model. For this reason, this paper outlines an investigation into the application of hierarchical clustering method to reduce the size of the problem. The efficiency of the proposed clustering algorithm is evaluated through a life of mine production scheduling optimization containing 2,487 drawpoints.

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2. LITERATURE REVIEW

2.1. Mathematical Programming Methods

Mathematical programming (MP) is the use of mathematical models, particularly optimizing models, to assist in making decisions. The MP model comprises an objective function that should be maximized or minimized while meeting some constraints that determine the solution space and a set of decision variables whose values are to be determined.

Eiselt and Sandblom [3] divide the modelling process in mathematical programming into eight steps: (1) problem recognition, (2) authorization to model, (3) model building and data collection, (4) model solution, (5) model validation, (6) model presentation, (7) implementation and monitoring, and (8) control.

The tractability of the mathematical models depends on the size of the problem, in terms of the number of variables and constraints, and the structure of the constraint sets. In the integer programming, as the size of an integer program grows, the time required for solving the problem increases exponentially. The most common problem in the mixed-integer linear programming formulation is the size of the branch and

cut tree. The tree becomes so large that insufficient memory remains to solve the LP sub-problems. The size of the branch-and-cut tree can be affected by the specific approach one takes in performing the branching and by the structure of each problem.

2.2. Block Caving

Block caving is an underground mining method appropriate for low-grade and massive ore-bodies. In this method, the ore is extracted from the bottom of the orebody to the top. First, the production level is developed below the orebody; then the orebody is undercut by blasting a layer of ore. After undercutting, the rock mass above starts to cave under its weight and in-situ stresses. The broken ore is extracted by load-haul-dump (LHD) machines from the drawpoints located in the production level. Above each drawpoint, a draw column is considered, and the material of the draw column is extracted from the relevant drawpoint (see Figure 1).

2.3. Production Scheduling Optimization in Block-Cave Mining

Using mathematical programming optimization with exact solution methods to solve the long-term

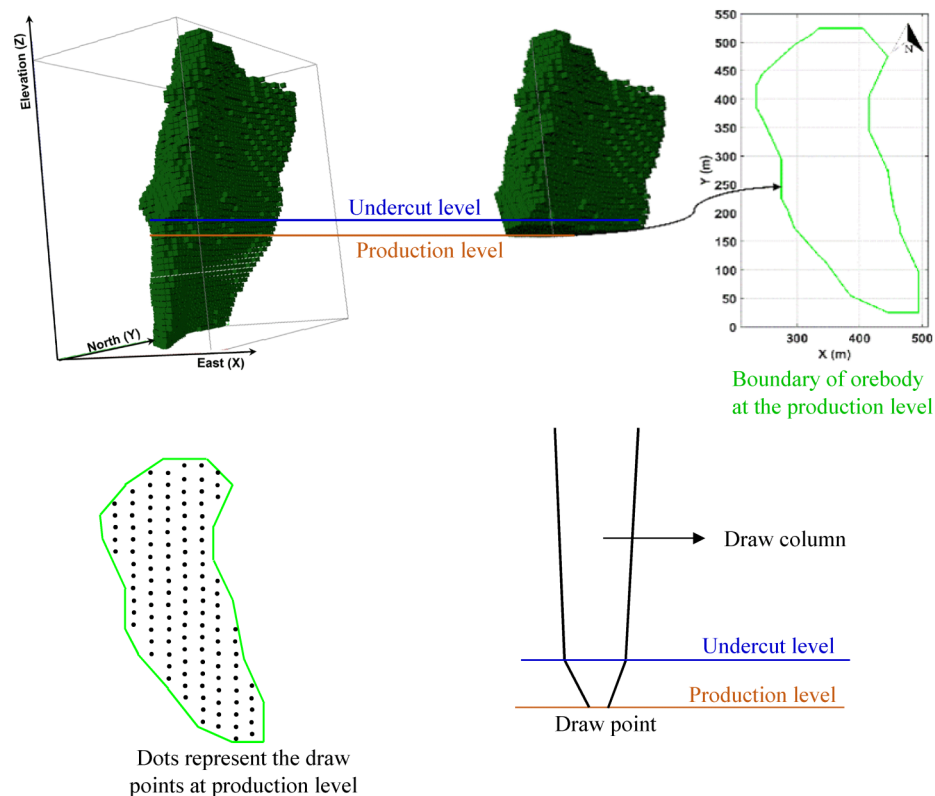


Figure 1: Schematic view of block caving production.

production planning problem has proved to be robust and results in answers within known limits of optimality. The mathematical programming models which are considered for production scheduling are linear programming (LP), mixed-integer linear programming (MILP), non-linear programming (NLP), dynamic programming (DP), multi-criteria optimization, network optimization, quadratic program (QP), and stochastic programming [4]. To optimize block-caving scheduling, most researchers have used mathematical programming [6-38]. Khodayari and Pourrahimian [5] presented a comprehensive review of operations research in block caving. In most of the available models, considering all the operational constraints will result in an oversize model and solving the model is time-consuming and exceeds the power of current computational systems.

2.4. Clustering

Clustering is defined as the process of grouping similar objects together in a way that maximizes intra-cluster similarity and inter-cluster dissimilarity. Hierarchical clustering procedures are among the best known statistical methods of clustering. One of the main aspects in clustering algorithms is determining of the similarity index for all objects which must be grouped. In addition to the similarity of constituent items, the generated clusters should consider some constraints such as minimum and maximum cluster sizes, limitation on the cluster shapes, and mutually exclusive and inclusive objects. Due to the low computational power required, clustering techniques are applied in mine planning programs.

Clustering classifies objects by conceptualizing principal configuration either as a grouping of individuals or as a hierarchy of groups. The cluster results can be studied and re-clustered if it does not satisfy preconceived ideas in the grouped data [39]. Clustering can be categorized into two major groups: hierarchical and partitional clustering. The hierarchical algorithm based on measured characteristics can create clusters progressively.

In the start, each object is a separate cluster, and then by combining the separated clusters together sequentially, the number of clusters at each grouping stage reduces. If there are N objects, this contains $N-1$ clustering stages [40]. In brief, the process of grouping similar entities together is the main clustering role. The clustering should cause to intra-cluster similarity maximize, and inter-cluster similarity minimize [40, 41].

Epstein *et al.* [13] applied aggregation in underground block-sequencing operations. Weintraub *et al.* [42] to reduce the size of the MIP models used an aggregation priori and a posteriori clustering methods based on a K-means algorithm. Tonnage, percentage of copper and molybdenum, and speed of extraction were the measuring tools of the similarity between clusters. Because of the different importance of each characteristic, a set of weights related with the characteristics was defined. According to Weintraub method, each cluster can be extracted only once, and the given sequence of extractions must be considered. Draw rate as one of the important parameters was not established as a characteristic to measure the dissimilarity between clusters. Newman and Kuchta [43] stated that, to overcome the size of MIP problem, aggregated time periods in the scheduling of an iron-ore underground mine. They used the information gained from the aggregated model to solve the original model. The original models involved 500 binary variables, while aggregated models, contained 260 binary variables. Pourrahimian *et al.* [4, 23] formulated a MILP model for block caving long-term production scheduling. They designed a hierarchical clustering method to overcome the size problem of mathematical programming models. Their model aimed to maximize the NPV of the mining operation at three different levels of resolution: (i) aggregated drawpoints (cluster level); (ii) drawpoint level; and (iii) drawpoint-and-slice level; in the model, the mine planner has control over defined constraints. They attempted to find an optimal schedule for the life of mine, solving simultaneously for all periods by considering all required constraints, but they did not consider the geotechnical properties of rock mass through the draw rate constraint. They noted that the formulation tried to extract material from drawpoints with a draw rate within the acceptable range without considering a specific shape.

A shortcoming of these methods is their dependency on the definition of similarity and their high sensitivity to the weights used in determining similarity. The proposed clustering algorithm in this paper is based on a hierarchical approach and is specifically developed to be used in solving block-caving mine production planning problem.

3. CLUSTERING ALGORITHM

Agglomerative and divisive clustering techniques are two distinct classes of hierarchical clustering algorithms. In the agglomerative technique (bottom up, clumping), there are n singleton clusters to start with,

and the process begins by successively merging clusters. The divisive technique (top down, splitting) puts all of the elements in one cluster and creates new clusters by sequentially splitting old clusters. The agglomerative procedure requires less computational effort compared to the divisive technique [40]. Aggregation techniques are highly dependent on the structure of the problem and tailored for a specific instance of a problem [43].

In the developed clustering algorithm, the portion of material scheduled to be extracted from each cluster is assumed to be taken from all of the active drawpoints, based on the ratio of each drawpoint's tonnage in the cluster.

Considering the direction of mining advancement in forming the clusters is a key strategy when dealing with mine economics or geotechnical problems. It is essential to develop a direction factor to be included in the similarity index to account for the mining advancement direction. For this purpose, the clustering technique developed by Tabesh and Askari-Nasab [44] was modified for its application in block-cave mining. According to Tabesh and Askari-Nasab, after defining the advancement direction, the engineer should specify two points at the starting and ending points in the direction of advancement. Afterwards, the direction factor can be calculated using Equation (1).

$$N_i = \text{Sign}\left((N_i^1)^2 - (N_i^2)^2\right) \times \sqrt{\left|(N_i^1)^2 - (N_i^2)^2\right|} \quad (1)$$

Where N_i^1 and N_i^2 are the distances from drawpoint i to start and end points respectively. The sign() function returns +1 if the value is positive and -1 if the value is negative.

The elements aggregation procedure needs a similarity measure or similarity index that quantifies the similarity between two objects. Various properties can be taken into account when defining similarities between draw columns. The multiple similarity indices algorithm are used in place of existing similarity index clustering models that contain weight factors defined by the planner.

Increasing the number of properties used in similarity calculations increases the complexity of the index, in terms of not representing a unique physical attribute. The developed algorithm aggregates the draw columns into clusters based on center-by-center distance, grade distribution, maximum draw rate according to the production rate curve (PRC), and

advancement direction. The general procedure of the proposed algorithm is as follows:

1. Define the number of required similarity indices according to the mining operation;
2. Define a search radius;
3. Each draw column is considered as a cluster. The similarities between clusters are the same as the similarities between the objects they contain in each index.
4. Define the maximum number of required clusters and the maximum number of allowed draw columns within each cluster for each index.
5. Similarity values are calculated for the considered similarity index.
6. The most similar pair of clusters is merged into a single cluster.
7. The similarity between the new clusters and the rest of the clusters is calculated. Steps 3 to 6 are repeated until the maximum number of clusters is reached or there is no pair of clusters to merge because the maximum number of allowed draw columns has been reached.
8. For the next similarity index, define an intra-cluster adjacency matrix for draw columns that are located within two different clusters.
9. Repeat steps 3 to 7 for the similarity index defined in step 8.

For the first step, the similarity index is calculated based on the distance and the most similar pair of clusters is merged into a single cluster (Equation (2)):

$$SI_1 = \frac{1}{D_{ij} \times N_{ij-SI_1}} \times A_{ij} \quad (2)$$

Where SI_1 is the similarity index of step 1 (distance), $\frac{1}{D_{ij} \times N_{ij-SI_1}}$ is the similarity value between draw columns i and j , D_{ij} is the normalized distance value between the center line of draw columns i and j , N_{ij-SI_1} is the normalized Euclidean distance between values N_i and N_j for SI_1 , and A_{ij} is the adjacency factor between draw columns i and j . If the distance is less than the defined search radius, A_{ij} is 1; otherwise, it is 0. Since the clustering is performed on the production level, the

z value of coordinates is not taken into account. This form the distance matrix for the production level. The matrix is then normalized by dividing all elements by the maximum value in the matrix.

The similarity between the new clusters and the rest of clusters is calculated. After calculating the similarity, the mentioned steps are repeated until the maximum number of clusters is reached or there is no pair of clusters to be merged, because the maximum number of allowed draw columns has been reached.

For the second step, similarity based on the maximum allowable draw rate, an intra-cluster adjacency matrix for draw columns that are located within two different clusters is required (Equation (3)):

$$SI_2 = SI_1 \times \frac{1}{MaxDR_{ij} \times N_{ij-SI_2}} \times ISA_{ij} \quad (3)$$

Where $SI_1 \times \frac{1}{MaxDR_{ij} \times N_{ij-SI_2}}$ is the similarity value between draw columns i and j , and ISA_{ij} is the second inter-cluster adjacency factor between draw columns i and j . If draw columns i and j are in the same cluster as they were in the first step, ISA_{ij} is 1; otherwise, it is 0.

In the third step, similarity is calculated based on the grade of draw columns (Equation (4)):

$$SI_3 = SI_1 \times \frac{1}{Grade_{ij} \times N_{ij-SI_3}} \times ITA_{ij} \quad (4)$$

Where $SI_1 \times \frac{1}{Grade_{ij} \times N_{ij-SI_3}}$ is the similarity value

between draw columns i and j , $Grade_{ij}$ is the normalized grade difference between draw columns i and j , and ITA_{ij} is the third intra-cluster adjacency factor between draw columns i and j . If draw columns i and j are in the same cluster as they were in the second step, ITA_{ij} is 1; otherwise, it is 0. This algorithm indirectly controls practical cave advancement.

4. ILLUSTRATIVE EXAMPLE

The production schedule of 2,487 drawpoints based on a defined PRC and cluster approach is investigated in this section. The total tonnage of material is 803.91 (Mt) with an average density of 2.2 (t/m³) and an average grade of 0.36%Cu. Figure 2 illustrates the tonnage and grade distributions of the draw columns.

For production scheduling, the MILP model presented by Nezhadshahmohammad and Pourrahimian [37] modified for cluster resolution and then used in this study. It should be noted that the original model was not able to solve the current problem because of the size that. The original model is at drawpoint level. The model was tested using a Dell Precision T7600 computer with Intel(R) Xeon(R) at 2.3 GHz, with 64 GB of RAM. The maximum depletion percentage of the drawpoints from the ramp-up to steady and steady to the ramp-down were 40% and 85%, respectively. A gap tolerance (EPGAP) of 5% was used as an optimization termination criterion. The

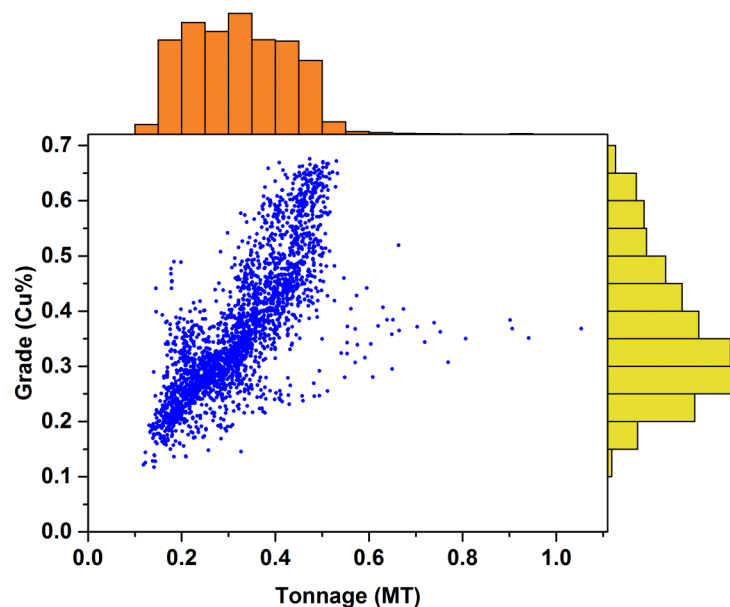


Figure 2: Tonnage (orange) and grade (yellow) distributions of draw columns. Each blue dot represents a draw column.

additional production scheduling parameters have been summarized in Table 1.

Table 1: Production Scheduling Parameters

Parameters	Value	
Maximum activity (periods)	5	
Mining capacity (Mt)	15 – 27.5	
Draw rate of draw columns (kt/period)	30 – 100	
Number of new clusters per period	0 – 11	
Production grade (%Cu)	0.3 – 0.6	
Number of maximum active clusters per period	25	
Max. number of clusters	First step	10
	Second step	40
	Third step	109
Max. number of draw columns	First step	350
	Second step	80
	Third step	25
Adjacency radius (m)	22	
Discount rate (%)	12	

Figure 3 shows the proposed clustering method for 2,487 drawpoints. The clustering was done in three steps. These steps were based on (i) the distance between drawpoints in the advancement direction, (ii) draw rate of drawpoints, and (iii) grade of draw columns. The advancement direction was determined based on the method presented by Khodayari and Pourrahimian [25]. The mining advancement direction in this case study is from south to north. The maximum number of clusters defined in the first, second, and third steps were 10, 40, and 109 respectively.

The problem was modelled both with and without clustering the drawpoints. The total number of constraints in the model without clustering was 610,548. The number of continuous and binary variables were 79,296 and 158,592 respectively. The model did not generate a solution after being run for 15 days.

The model with clusters comprised of 30,516 constraints and the number of continuous and binary variables were 3,488 and 6,976 respectively. Using the multi-similarity index aggregation technique resulted in a 95% reduction in the number of binary variables. The clustered model was solved in 37.8hrs. 761.87Mt of ore

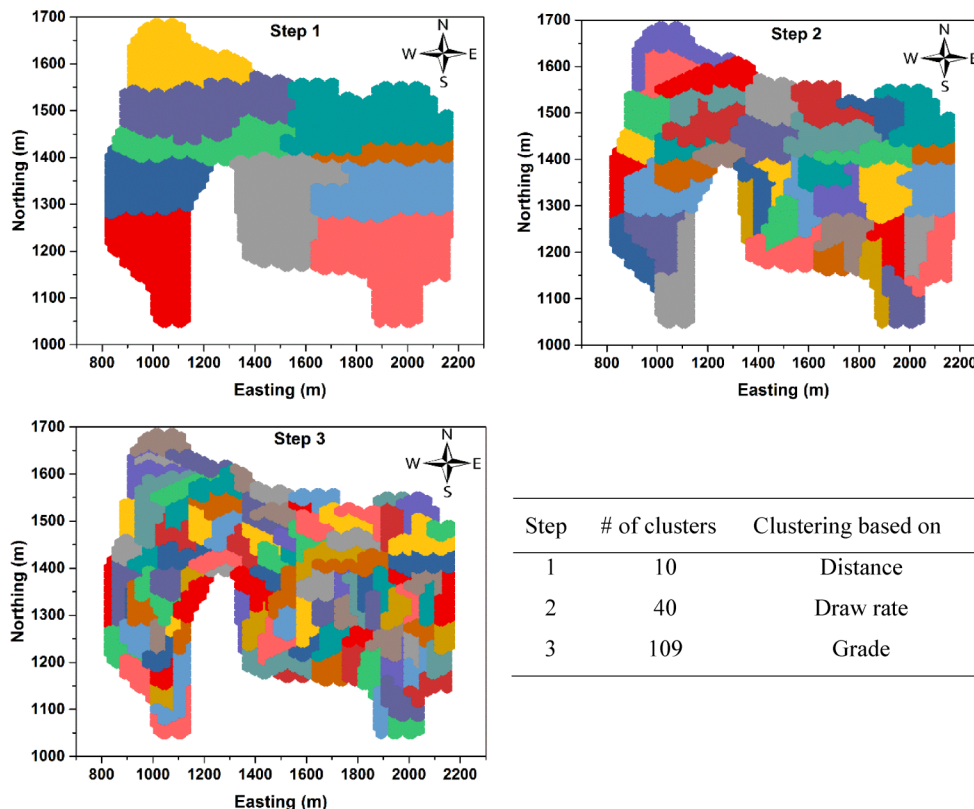


Figure 3: Application of the proposed clustering algorithm for 2,478 draw columns.

was extracted during 32 years of production generating NPV of \$304.6 B.

Figure 4 shows the cash flow, tonnage mined, and average grade of production in each period for the clustered model. The required ramp-up and ramp-down for the total production mine life is achieved in the resulting production plan.

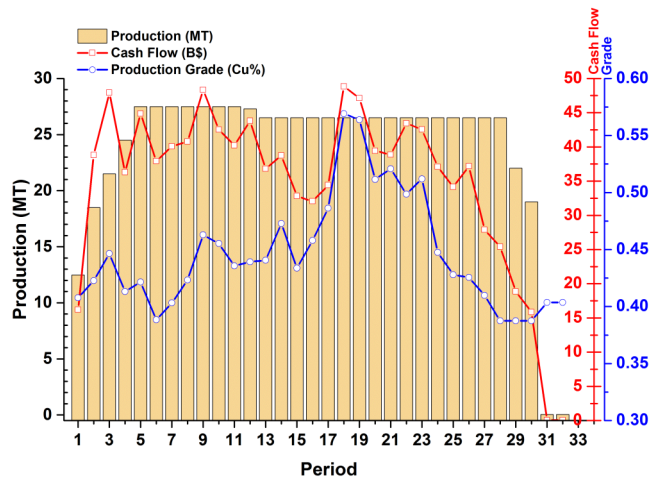


Figure 4: Annual cash flow, tonnage mined and average grade of production from the clustered model.

Figure 5 shows the grade distribution in the clusters. Figure 6 illustrates the starting period of extraction from cluster during the mine life. The starting period of drawpoints shows the advancement direction of caving achieved through production scheduling optimization.

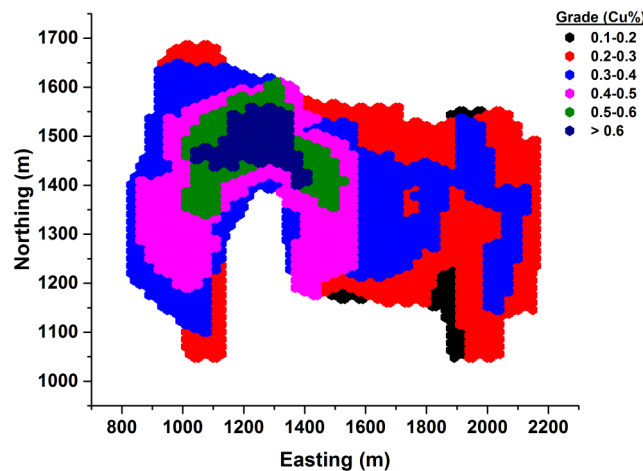


Figure 5: Grade distribution in the clustered model.

Following the defined sequence of extraction, the high-grade clusters are extracted during periods 15 to 21 (Figure 6). As a result, the grade of mine production increases during that period of time (Figure 4). Because of the application of the production rate curve

for draw control, it is expected there will be less dilution during the mine life.

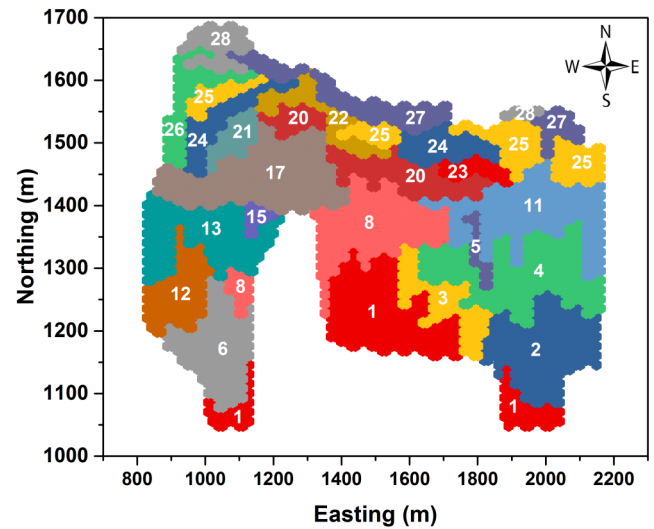


Figure 6: Starting period of different areas over the mine life.

Figure 7 shows the maximum number of active clusters and number of new clusters which had to be opened in each period. The number of active clusters in period 1 is equal to the number of new clusters which opened. From periods 12 to 15, this number gradually reduces. The number of new clusters opened in period 1 could be equal to the maximum allowable number of active clusters to reach the required production in this period. There was no need to open new clusters in periods 7, 9, 10, 14, 16, 18, 19, 29, 30, 31, and 32.

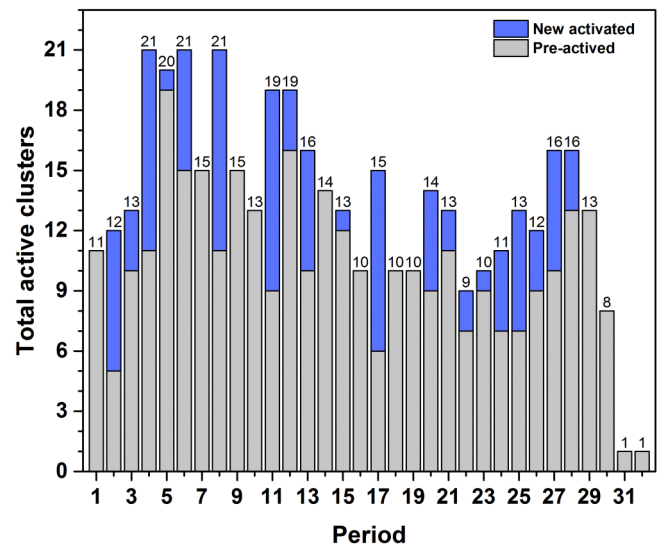


Figure 7: Number of active and new clusters in the model.

The results show that all the defined constraints have been satisfied. The starting and finishing periods,

and draw rates for each cluster are outputs of the optimization. The model extracts the material from each cluster based on the defined PRC model while maximizing the NPV of the operation.

5. CONCLUSION

This paper presented an aggregation approach to use in block cave production scheduling. Because of the size of the problem, the model could not be solved within a reasonable time. The clustering techniques implemented resulted in a 95% reduction in the number of binary variables which made it possible to solve the same problem in an acceptable CPU time. The solution time would enable the mine planner to analyze different scenarios during the feasibility studies. The presented aggregation approach eliminates dependency on the weighting factor in the current clustering techniques; as a result, reducing the effect of human errors on the optimality of the production schedule. Using the presented clustering approach with the mathematical formulations, the life of mine production schedule for large-scale block caving operations can be optimized.

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Received on 30-12-2018

Accepted on 5-3-2019

Published on 6-3-2019

DOI: <http://dx.doi.org/10.15377/2409-5710.2018.05.4>

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