A New Model for Accurate Prediction of Pressure Drop in Gas Well

Fadairo Adesina[1] and Olafuyi Olalekan[2,*]

[1]Department of Petroleum Engineering Covenant University, Ota, Ogun State, Nigeria

[2]Department of Petroleum Engineering, University of Benin, Benin City, Edo State, Nigeria

Abstract: The analytical model for predicting the pressure at any point in a flow string from the bottom hole to the well head is essential in determining optimum production string dimension and in the design of gas-lift installations.

A variety of models for predicting pressure transience in flowing wells have been reported in the literatures. Most of the early models were based on steady state fluid flow equation that did not consider time factor which results in inaccurate at early production time. Hence, there is the need for developing a model capable of estimating pressure transverse accurately at all times in flowing well bore.

This study presents equation for pressure drop in flowing vertical well without neglecting any term in the momentum equation by the inclusion of accumulation and kinetic term. The analytical solution of the resulting differential equation gives functional relationship between flow rates and pressure at any point in flowing well at any given time. The results show improvement over previous studies as the assumptions previously neglected were all considered.

Keywords: Pressure drop, gas well, transient phenomena, pressure losses.

INTRODUCTION

Bottom hole pressure is the pressure measured in a well at or near depth of the producing formation. This pressure is usually measured in pounds per square inch (psi), at the bottom of the hole. Bottom hole pressure is used to represent the sum of all the pressures being exerted at the bottom of the hole [1-10].

There is nothing more important in petroleum engineering than a definite knowledge of the pressure at the bottom of a well at any operating condition, and the relation of this pressure to the pressure within the producing formation. The ability of a gas reservoir to produce for a given set of reservoir conditions depends on the flowing bottom hole pressure and can be mathematically expressed as [9]

$$q = c(P_R^2 - P_{wf}^2)^n \tag{1}$$

From a gas well test data, plotting q against $(P_R^2 - P_{wf}^2)^n$ on a log-log graph, information that can be obtained from this plot is the absolute open flow potential (AOFP) of the well.

For reservoir engineering calculations the static bottom-hole pressure is frequently required. For a shut-in well, the flow rate is equal to zero we have a general equation:

$$\int_{Pwh}^{Pws} \left(\frac{ZT}{p} dp \right) = 0.01875 \gamma_g z \tag{2}$$

In many cases, it may be difficult or expensive to obtain static bottom-hole pressure value by gauge measurement; techniques have been made to calculate static bottom-hole pressure from wellhead pressure measurements. Several correlations are presently available, but, the best known are: the average temperature and z-factor method, the Cullender and Smith method, the Sukkar and Cornell method

The bottom-hole pressure is the major factor that dictates the production rate and information, and it is needed at all time during the life of the well. If the pressure profile can be predicted accurately, then the achievable production rate from a well can be projected with large accuracy. It can also be used to determine the inflow performance relationship of the reservoir. Several correlations have been developed for the prediction of bottom-hole pressure in single phase vertical pipe; some neglected the effect of turbulence and others neglected the fact that friction factor cannot be constant throughout the length of the flow string. While some considered the aforementioned factors, others assumed a constant average value of both temperature and gas deviation factor. The major problem with large numbers of early correlations is that it involves the use of trial and error procedure to solve the equation at some depth below the surface, hence can be tiring when large data are to be computed. But Poettmann [6] came up with a correlation to predict the pressure drop in static well that is fairly accurate and easier to use because it does not involve trial and error

*Address correspondence to this author at the Department of Petroleum Engineering, University of Benin, Benin City, Edo State, Nigeria;
E-mail: abdulkashif@yahoo.com

E-ISSN: 2409-787X/14

method. It assumed that temperature is constant at an average value over the entire length of the flow string. This assumption gives accurate result for shallow wells but it is not valid for wells with great depth. Sukkar and Cornell [7] improved on Poettmann [6] correlation by integrating numerically at several constant average temperatures. This correlation has the advantage of improved accuracy and does not require the trial and error procedure for the calculation of bottom-hole pressure. Cullender and Smith [8] extended Sukkar and Cornell [7] method by calculating gas deviation factor as a function of both temperature and pressure. The major setback of this model is that it is tedious and time consuming. Messer and Raghavan [9] extended the Sukkar and Cornell [7] method by applying it to slanted wells at reduced temperatures of up to 3.0 and reduced pressures of up to 30. Guo [10] modified the constant average temperature and average gas deviation factor method by replacing the constant friction factor in the general equation for fully turbulent flow in rough pipes. Adekomaya *et al.* (2011) [11] modified the Sukkar and Cornell [7] method by introducing a friction factor as a function of diameter. Fadairo *et al.* [12] adapted Sucker and Cornel concept and extend their model to estimate pressure drop during cutting transport and cleaning in the wellbore. None of these previous authors considered accumulation and kinetic terms in the fundamental energy equation, used for pressure drop model derivation that is very significant at the onset of gas production. The new model is capable to estimate the pressure drop in gas flowing well in all type of flow (turbulent, laminar or transitional), at all production time. The new concept revealed that the act of considering all pressure losses term in pressure drop estimation for gas well gives a more realistic result that include the initial unsteadiness phenomenon hence predict productivity at any given production time.

MODEL DEVELOPMENT

The model is developed to help predict the flowing bottom-hole pressure as a sum of all the pressure losses and the tubing head pressure. These pressure losses are; pressure drop due to friction plus pressure drop due to kinetic energy plus pressure drop due to potential energy plus pressure drop due to accumulation.

This new model is taking into consideration the fact that kinetic term and accumulation term have effects on the pressure drop and thus not to be neglected. The basic assumptions made in this study are;

1. The temperature is assumed constant at some average value
2. Flow could either be turbulent, laminar or transitional
3. No external work is done on the system

MODEL DEVELOPMENT

In the development of model, the basic energy equation reported as [1, 2];

$$144\frac{1}{\rho}dp + \frac{gdz}{gc} + \frac{fu^2}{2gcD}dL + \frac{udl}{2gcdt} + \frac{u^2}{2 \propto gc} = 0 \qquad (3)$$

Where the velocity (u) is given as

$$u = \frac{0.4152qTz}{PD^2} \qquad (4)$$

Therefore; equation (3) becomes,

$$144\frac{1}{\rho}dp + \frac{gdZ}{gc} + \frac{f\left(\frac{0.4152qTZ}{PD^2}\right)^2}{2\,\mathrm{gc}\,D}dL + \frac{\frac{0.4152qTZ}{PD^2}dl}{2\,\mathrm{gcd}\,t} + \frac{\left(\frac{0.4152qTz}{PD^2}\right)^2}{2gc} = 0 \qquad (5)$$

Density can be defined as [10]

$$\rho = \frac{29\gamma_g P}{zRT} \qquad (6)$$

Substituting equation (6) in equation (5), we have

$$144\frac{1}{\frac{29\gamma_g P}{zRT}}dp + \frac{gdZ}{gc} + \frac{f\left(\frac{0.4152qTz}{PD^2}\right)^2}{2\,\mathrm{gc}\,D}dL + \frac{\frac{0.4152qTz}{PD^2}dl}{2\,\mathrm{gcd}\,t} + \frac{\left(\frac{0.4152qTz}{PD^2}\right)^2}{2gc} = 0 \qquad (7)$$

Expanding equation (8)

$$\frac{53.28}{zT}dp + dl + \frac{f}{2gcD}\left(\frac{0.1724q^2T^2Z^2}{P^2D^4}\right)dL + \left(\frac{0.4152qTz}{PD^2}\right)\frac{dL}{2\,\mathrm{gcd}\,T} + \frac{\frac{0.1724q^2T^2Z^2}{P^2D^4}}{2 \propto gc} = 0 \qquad (8)$$

Given gc as 32.17 and inserting into equation (8), we have,

$$\frac{53.28}{zT}dp + dl + \frac{f}{64.34}\left(\frac{0.1724q^2T^2Z^2}{P^2D^5}\right)dL + \left(\frac{0.4152qTz}{PD^2}\right)\frac{dL}{64.34dT} + \frac{\frac{0.1724q^2T^2Z^2}{P^2D^4}}{64.34} = 0 \quad (9)$$

Converting all units to field units and re-arranging, we have;

$$\frac{53.28}{zT}dp + dl + f\left(\frac{667q^2T^2Z^2}{P^2D^5}\right)dL + \left(\frac{0.9295qTZ}{PD^2}\right)\frac{dL}{dT} + \frac{55.56q^2T^2Z^2}{P^2D^4} = 0 \quad (10)$$

Collecting like terms and integrating, we have

$$\int_{P_{tf}}^{P_{wf}} \frac{\frac{z}{Ppr}dppr}{1 + C_1 f\left(\frac{z}{Ppr}\right)^2 + C_2\left(\frac{z}{Ppr}\right) + C_3\left(\frac{z}{Ppr}\right)^2} = \frac{(\mathbf{0.01875})(\gamma g)(\boldsymbol{Z})}{\boldsymbol{T}} \quad (11)$$

Separating the variables with a constant lower limit, the lower limit will cancel out when integrating over two intervals, we have

$$\int_{12}^{P_{wf}} \frac{\frac{z}{P_{pr}}dp_{pr}}{1 + C_1 f\left(\frac{z}{P_{pr}}\right)^2 + C_2\left(\frac{z}{P_{pr}}\right) + C_3\left(\frac{z}{P_{pr}}\right)^2} - \int_{12}^{P_{tf}} \frac{\frac{z}{Ppr}dp_{pr}}{1 + C_1 f\left(\frac{z}{P_{pr}}\right)^2 + C_2\left(\frac{z}{P_{pr}}\right) + C_3\left(\frac{z}{P_{pr}}\right)^2} = \frac{(\mathbf{0.01875})(\gamma g)(\boldsymbol{Z})}{\boldsymbol{T}} \quad (12)$$

Hence;

$$\int_{12}^{\mathrm{P_r}} \frac{\frac{z}{P_{pr}}dpr}{1 + C_1 f\left(\frac{z}{\mathrm{P_r}}\right)^2 + C_2\left(\frac{z}{\mathrm{P_r}}\right) + C_3\left(\frac{z}{\mathrm{P_r}}\right)^2} = \frac{(\mathbf{0.01875})(\quad g)(\boldsymbol{Z})}{\boldsymbol{T}} \quad (13)$$

Where;

$$C1 = \frac{667Fq^2T^2}{D^5 P_{pc}{}^2} \quad C2 = \frac{(0.9292)(q)(T)}{D^2 P_{pc}(t)} \quad C3 = \frac{(55.55959)(q^2)(T^2)}{(L)(D^4)(P_{pc}{}^2)} \quad (14)$$

Equation 14 is numerically integrated to obtain the value of the integral. An excel macros program was developed to solve the integral given the upper limit using trapezoidal rule and an interval of equal spacing.

Trapezoidal rule states that;

$$\int_a^b y\,dx \approx (widthofinterval)\left[\frac{1}{2}(first + lastordinate) + (sumofremainingordinates)\right] \quad (15)$$

In order to ensure accuracy in the course of this study, Chen correlation was used to obtain the friction factor, while Lucas correlation was used to obtain viscosity. Other pressure dependent variables considered such as gas density, compressibility factor and formation volume factor equations are generated from the existing correlations in the literatures and reported respectively as follows

The gas density can be determined from the general gas law at reservoir conditions as follows:

$$\rho_g = \frac{P\gamma_g M_{air}}{ZRT} \quad (16)$$

The gas-compressibility (Z-factor) can be determined by the Dranchuk and Abou-Kassem (DAK) correlation (1975). The error between the Z–factor obtained from the Standing and Katz (1942) charts and the Z-factor determined by use of the DAK correlation was less than 0.0001. The correlation used for Z-factor calculation can be written as follows:

$$Z = 1 + \left(A_1 + \frac{A_2}{T_{pr}} + \frac{A_3}{T_{pr}{}^3} + \frac{A_4}{T_{pr}{}^4} + \frac{A_5}{T_{pr}{}^5}\right)\rho_{pr} + \left(A_6 + \frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}{}^2}\right)\rho_{pr}{}^2 - A_9\left(\frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr^2}}\right)\rho_{pr}{}^5 + A_{10}\left(1 + A_{11}\rho_{pr}{}^2\right)\left(\rho_{pr}{}^2/T_{pr}{}^3\right)EXP\left(-A_{11}\rho_{pr}{}^2\right) \quad (17)$$

$$\rho_{pr} = 0.27\left(\frac{P_{pr}}{zT_{pr}}\right) \quad (18)$$

$$P_{pr} = \frac{P}{Pc'} \qquad T_{pr} = \frac{T}{Tc} \quad (19)$$

The values of the constants $A_1\,through\,A_{11}$ are listed in the Table **1** below

The Z-factor was estimated by first assuming a value for Z–factor, and then the reduced density (ρ_{pr}) was determined by use of equation (16). Finally, Z-factor was determined by use of equation (17) while

Table 1: Showing the Constant Values in Dak Correlation

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
0.3265	-1.07	-0.534	0.01569	-0.052	0.5475	-0.736	0.1844	0.1056	0.6134	0.721

ρ_{pr} and T_{pr} were determined by use of the corrected critical pressure (P_{pc}) and critical temperature (T_{pc}).

The formation volume factor is defined as [1,12]

$$\int_{12}^{pr} \frac{\left(\frac{z}{p}\right)}{1 + C_1\left(\frac{z}{p}\right)^2} \partial \mathbf{Pr} \tag{20}$$

This study considered the effect of kinetic and accumulation term on the fundamental momentum equation used in the formulation of pressure drop along wellbore which was unaccounted for in the previous studies in the literature**.** The difference in the results obtained from this study and that of Suker and Cornel model has shown that these two terms have significant effect which cannot be overlooked as opined by the earlier authors.

The Suker and Cornel model among other models can only be valid for steady state flow at the later production time, whereas, this is not always the case in real scenario. It may also be realistic if the pipe length is short and all pressure dependant variations can be assumed constant.

The models that predict pressure drop at different depth in the wellbore during gas production reported by Suker and Cornel and the present study are given as equations 23 & 25 respectively.

Sukkar and Cornell developed the model below.

$$C_1 = \frac{667Fq^2T^2}{D^5 P_{pc}{}^2}, C_2 = \frac{(0.9292)(q)(T)}{D^2 * P_{pc} * t}, C_3 = \frac{(55.55959)(q^2)(T^2)}{(L)(D^4)(P_{pc}{}^2)} \tag{23}$$

$$C_1 = \frac{667 f q^2 T^2}{D^5} \tag{24}$$

For this study, the final model for expressing pressure transience in wellbore is given as

$$\int_{12}^{pr} \frac{\frac{z}{p_{pr}} d_{pr}}{1 + C_1 f\left(\frac{z}{p_r}\right)^2 + C_2\left(\frac{z}{p_r}\right) + C_3\left(\frac{z}{p_r}\right)^2} = \frac{(\mathbf{0.01875})(\Upsilon g)(\mathbf{Z})}{\boldsymbol{T}} \tag{25}$$

$$C_1 = \frac{667Fq^2T^2}{D^5 P_{pc}{}^2}, C_2 = \frac{(0.9292)(q)(T)}{D^2 * P_{pc} * t}, C_3 = \frac{(55.55959)(q^2)(T^2)}{(L)(D^4)(P_{pc}{}^2)} \tag{26}$$

Using the Sukkar and Cornell data from the literature as reported in Table **2**, the left hand side of the developed model was solved by numerical integration. The choke pressure (P_s) is known as it is estimated from the surface while the bottom-hole pressure (P_b) is unknown; the point of focus is in this regard.

This new method is capable of yielding a satisfactory pressure differential result during flow of gas at point in the wellbore, at all time and at both steady and unsteady state period. All pressure dependent variables are treated as a function of pressure and not a constant as opined by many investigators.

The following parameters have been used to validate this model [7]

Table 2: Input Data (Fluid and Pipe Parameters)

Sg	0.6
Z=L	5790
Tavg	577
Tr	1.61
Pwh	2300
Ppc	672
D	2.259
q	5
LHS	0.11289
Pr	3.422619

Figure **1** shows the effect of inclusion of kinetic and accumulation terms in the fundamental momentum equation for predicting pressure drop in a flowing gas well. It reports the variation of pressure decline in a flowing vertical gas well with production time. The figure depicts that the pressure drop decreases from 0 to approximately 20days and then stabilizes above 20days of production time. The difference in pressure drop with frictional loss only and pressure drop with all possible losses is the amount of flow restricted by both

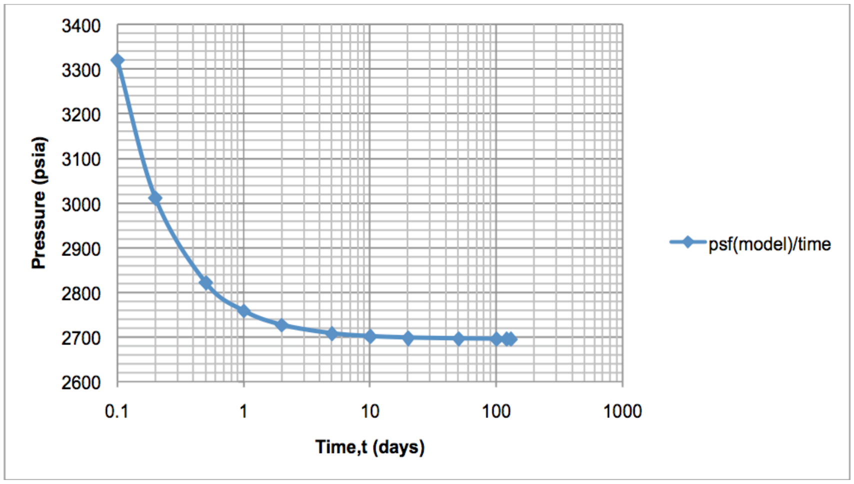

Figure 1: Transient curve showing the pressure transverse against production time.

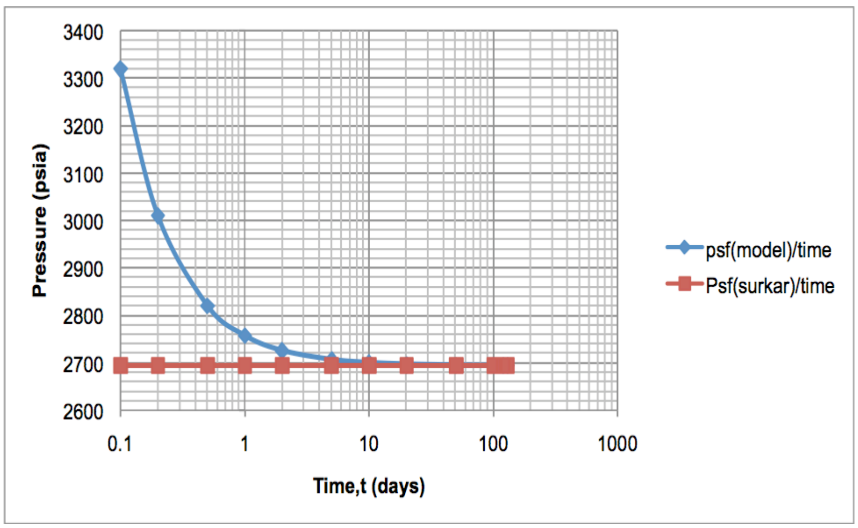

Figure 2: Transient curve showing the pressure transverse against production time between Sukkar and Cornell model and the new model.

kinetic energy change and fluid accumulation. This difference is less significant at the later time of production. Thus, it is evident that there exists an initial transience at the onset of flow which later stabilizes with time.

Figure **2** compares the existing models (Suker and Cornel) with the modified model at the early stage of production time. This implies that at this stage, Suker and Cornel might had under-estimated pressure drop for failure to consider wellbore pressure losses due to kinetic change and fluid accumulation. At the onset of production, the effect of all possible wellbore pressure losses is highly pronounced and increased with time as the vertical well length increases.

CONCLUSION

The newly developed model has been verified to be more accurate than the Sukkar and Cornell model. The newly developed model takes into consideration the effect of the type of flow, the effect of varying viscosity as well. The only variable that was assumed to be constant was the temperature.

Sukkar and Cornell can still be applied at very shallow depths, since the effect of the kinetic energy is negligible in such ranges. The newly developed model however can be used at all depths. The effect of using the Sukkar and Cornell model is extremely adverse for calculation of other parameters such as flow rate and carrying out economic analysis.

Therefore it can be concluded that an analytical model for estimating pressure drop in vertical flowing gas wellbore is developed without neglecting any of the terms in the fundamental governing differential equations for gas well.

Consider all pressure losses term in pressure drop in vertical flowing gas wellbore, a more realistic result that include the initial unsteadiness phenomenon hence predict pressure transient at any given production time.

At the onset of production, the effect of all possible wellbore pressure losses is highly pronounced and increased as the vertical well length increases.

RECOMMENDATION

- Kinetic term and accumulation term should be considered when calculating bottom-hole pressures of a flowing well.
- Verification of this model should be done with at least two or three field data to further validate its accuracy.
- In terms of the degree of accuracy required, it should be noted that the overall accuracy of the model is subject to the measurement of gas rate, measurement of flowing wellhead pressure and temperature, measurement of natural gas specific gravity. These variables, if not properly estimated may subject the model interpretation to apparent error.
- Further research work is needed, however in terms of estimating the flowing bottom hole pressure of different flow regimes at different flow type in multiphase flow system. Although this seems to be out of scope for this study.

NOMENCLATURE

A = Cross-sectional area of pipe, ft^2

B = Formation volume factor

C = Numerical coefficient

dL = Incremental depth, ft

dp = Pressure differential, $\frac{lb}{ft^3}$

f_m = Moody friction factor, dimensionless

$\frac{fu^2}{2g_C D}dL$ = Pressure drop due to friction effects

g = Acceleration due to gravity, $\frac{ft}{sec^2}$

g_c = Conversion factor, 32.17 $\frac{lbmft}{lbfs}$

L = Length of the flow string, ft (for vertical flow string, L=Z)

M_{air} = Molecular weight of air, $29\frac{lbm}{lbm}mol$

N_{Re} = Reynolds Number

n = Numerical exponent, characteristic of a particular well

P = Bottom-hole Pressure, psia

P_R = Shut-in reservoir pressure

P_{wf} = Flowing bottom-hole pressure

q = gas flow rate

R = Gas constant, 10.73 $\frac{ft^3 psia}{lb-mole^\circ R}$

T = Temperature, ^OR

V = Specific volume of fluid, $\frac{ft^3}{lbm}$

Vg = Gas velocity, ms^{-1}

Ws = Mechanical work done on system

Z = Gas compressibility factor, dimensionless

ρ_g = Gas density, $\frac{lbm}{ft^3}$

μg = Gas viscosity, cp

REFERENCES

[1] Fadairo A, Femi-Oyewole D, Falode O. An Improved Tool for Liquid Loading In a Gas Well, SPE 167552-MS, August 2013, Nigeria.

[2] Fadairo AS, Ako C, Falode O. Modeling Productivity Index for Long Horizontal Well. J Energy Resour Technol 2011; 133(3).

[3] Barrufet A, Rasool A, Mohammed. Prediction of Bottom-hole Flowing Pressure in Multiphase Systems Using Thermo-dynamic Equation of State, SPE 29479 Presented at the 1995 SPE Production Operation held in Oklahoma April 2-4, 1995.

[4] Osman EA, Ayoub MA, Aggour MA. Artificial Neural Network Model for Predicting Bottom-hole Flowing Pressure in Vertical Multiphase Flow. Paper SPE 93632, Presented at the 14th SPE Middle East Oil and Gas show and Conference held in Bahrain International Exhibition Centre, Bahrain, 12-15 March 2005. http://dx.doi.org/10.2118/93632-MS

[5] Rzasa MJ, Katz DL. Calculation of static pressure gradients in gas wells [J]. Trans AIME 1945; 160: 100-113. http://dx.doi.org/10.2118/945100-G

[6] Poettmann FH. The calculation of pressure drop in the flow of natural gas through pipe [J]. Trans AIME 1951; 192: 317-326.

[7] Sukkar YK, Cornell D. Direct calculation of bottom hole pressures in natural gas wells [J]. Trans AIME 1955; 204: 43-48.

[8] Cullender MH, Smith RV. Practical solution of gas-flow equations for wells and pipelines with large temperature gradients. SPE 000696 1956.

[9] Messer PH, Raghavan R, Ramey HJ, Jr. Calculation of bottom-hole pressures for deep, hot and sour gas wells [J]. J Petrol Technol 1974; 26(1): 85-92. http://dx.doi.org/10.2118/3913-PA

[10] Guo BY. An analytical model for gas-water-coal particle flow in coalbed-methane production well. The proceedings of SPE Eastern Regional Meeting. Canton: Society of Petroleum Engineers 2001.

[11] Adekomaya AO, Adewuyi A, Olafuyi O. Effect of turbulent flow on pressure drop in a single phase vertical pipe [J]. Open Petrol Eng J 2011; 4: 1-8. http://dx.doi.org/10.2174/1874834101104010001

[12] Fadairo AS, Adekomaya O, Falode O. Effect of drilling cuttings transport on pressure drop in a flowing well, The proceedings of SPE Eastern Regional Meeting. Manama: Society of Petroleum Engineers 2009.

[13] Katz DL, Lee RL. Natural gas engineering–production and storage [M]. New York: McGraw Hill Publishing Company, 1990.

Received on 31-07-2014 Accepted on 22-08-2014 Published on 17-11-2014

DOI: http://dx.doi.org/10.15377/2409-787X.2014.01.01.3