Published by Avanti Publishers

International Journal of Petroleum Technology

ISSN (online): 2409-787X


# Risk-Adjusted Real Options Valuation for Petroleum Investments: An Expected Utility–Based Decision Framework

Rouholah Ahmadi*

*National Iranian Oil Company (NIOC), Ahvaz, Iran*

## ARTICLE INFO

*Article Type*: Research Article

*Academic Editor*: Mohammad Islam Miah

*Keywords*:
Risk-neutral
Certainty equivalent
Managerial flexibility
Expected utility theory
Decision tree modeling
Risk-averse decision-maker
Oil and gas investment strategies

*Timeline*:
Received: August 29, 2025
Accepted: October 10, 2025
Published: November 08, 2025

*Citation*: Ahmadi R. Risk-adjusted real options valuation for petroleum investments: An expected utility–based decision framework. Int J Pet Technol. 2025; 12: 108-132.

*DOI*: https://doi.org/10.15377/2409-787X.2025.12.8

## ABSTRACT

Despite the growing use of decision analysis in oil and gas investments, the industry continues to underperform in generating consistent value. Conventional valuation methods often treat uncertainty as a risk to be minimized rather than an opportunity to be exploited, overlooking the strategic role of managerial flexibility, or real options. At the same time, they typically assume risk neutrality, which misrepresents the industry's inherently risk-averse decision-making culture. Common risk measures such as value-at-risk fail to capture true preferences, leading to distorted valuations and suboptimal decisions.

This study addresses these limitations by integrating Real Options Valuation (ROV) with Expected Utility Theory (EUT) to jointly incorporate flexibility and risk preferences into petroleum project evaluation. The contribution lies in a fully operational workflow that links influence-diagram structuring, probabilistic uncertainty modelling, and certainty-equivalent valuation within a recombining binomial decision tree. The framework is applied to a representative oil production project featuring divestment and buyout options. Risk aversion is represented through an exponential utility function, with project values expressed as certainty equivalents reflecting subjective risk tolerance.

Findings show risk preferences fundamentally alter value and optimal strategy. A risk-neutral decision-maker values the project at $280.9 million, while one with a $20 million risk tolerance values it at $200.2 million. Strategically, the risk-neutral agent continues operations in 50% of scenarios, whereas the risk-averse agent divests in 75%. Sensitivity analysis identifies a tipping point near 30 million USD, beyond which choices converge toward risk neutrality.

Overall, the proposed ROV–EUT framework delivers realistic, decision-consistent valuations and clarifies how flexibility protects downside exposure for conservative firms while enabling opportunity capture for more risk-tolerant ones.

*Corresponding Author

Email: rouholah.ahmadi@gmail.com
Tel: +(98) 9394054352

# 1. Introduction

Investments in petroleum exploration and production are characterized by high capital intensity, long lifecycles, and exposure to multiple sources of uncertainty, including geological heterogeneity, reservoir performance, oil and gas price volatility, environmental regulations, and technological risks. These uncertainties can significantly affect the economic viability of projects and create challenges for decision-makers (DMs) tasked with allocating capital under uncertainty. Traditional discounted cash flow (DCF) analysis has long been the standard evaluation method in the petroleum industry, but it is inherently static and assumes a fixed path of development. As a result, DCF often underestimates project value because it ignores the flexibility that managers have to adapt decisions over time [1, 2].

In practice, petroleum projects involve a series of staged decisions — such as exploration drilling, appraisal, development, expansion, or abandonment — which can be revised as new information becomes available. These decision points embody what is often referred to as "managerial flexibility" or "real options." Real options valuation (ROV) explicitly recognizes this flexibility, allowing DMs to capture upside potential while mitigating downside risks [3-5]. Recent applications in petroleum engineering highlight the usefulness of ROV in field development planning, exploration portfolio optimization, and production management [6, 7].

Despite this progress, much of the literature has assumed that DMs are risk-neutral, valuing projects based only on expected monetary outcomes. In reality, however, DMs and organizations in the petroleum industry are often risk-averse, reflecting concerns about potential downside losses, capital exposure, and reputational risk [8]. Industry metrics such as value-at-risk (VaR) or probability of success (PoS) have been adopted to approximate risk attitudes, but they often provide inconsistent or biased recommendations [9]. Expected Utility Theory (EUT), originally developed by von Neumann and Morgenstern (1947), offers a consistent and rigorous framework for modelling DMs' preferences under uncertainty [10]. By applying utility functions — particularly the exponential form — EUT can account for different risk attitudes in a transparent and mathematically sound way [11, 12].

A number of studies have examined the role of risk attitudes in petroleum decision analysis. Costa Lima and Suslick (2008), for example, showed that incorporating risk preferences can substantially alter project valuations and investment recommendations [13]. More recently, attention has turned toward integrating modern valuation methods with practical decision-support tools for petroleum engineers and geoscientists. Ahmadi and Bratvold (2023), among others, have emphasized the applicability of real options in petroleum engineering contexts [7], while still leaving open questions on how to explicitly integrate DMs' risk preferences into the valuation of managerial flexibility.

Despite extensive academic development of real-options and utility-based decision models, practitioners in oil and gas rarely use them. Industry studies and recent surveys repeatedly report that managers and engineers default to single-case net present value (NPV) estimates or simple Monte Carlo simulations (MCS), viewing real options as too "complicated" or abstract [14, 15]. As one upstream specialist admitted, "the reliance on one number is hard to get away from… even at board level, they don't tend to deal with numbers for the ranges" [16], and the idea of reporting a value range can seem "completely alien" to field engineers [17]. In practice, few even learn to use preference models: one interview study found only four respondents had ever heard of "preference theory" and none actually applied it, noting it would be "difficult to convince the hard-nosed asset manager" to use such an approach [15]. In short, decision analysts conclude that "interesting" tools like real options and multi-attribute utility have failed to catch on because of entrenched habits – most companies report that "Monte Carlo and decision trees are about as far as it goes" [14] – and senior leaders expect a crisp single forecast.

Several authors have highlighted the cultural and practical barriers reinforcing this gap. Oil-industry managers tend to be conservative and risk-averse, with a short-term mindset and "fear of change" built into organizational culture [15]. Moreover, many petroleum engineers receive little training in probabilistic decision making: undergraduate curricula emphasize finding the one "right answer," so quantifying uncertainty feels "alien" [17]. Real-options modeling also suffers from a steep learning curve and communication hurdle – experts describe it as a "black box" that few executives understand [14]. Aspinall *et al*. (2023) confirm that even commodity projects (mining, energy) still default to static DCF models; firms only use ROV when experts are available, often outsourcing the work, and cite "lack of expertise" and complexity as primary deterrents [14]. Together, these factors – organizational inertia, demand for deterministic answers, limited training in uncertainty, and the perceived complexity of advanced methods – help explain why real options and EUT, though championed in the literature, remain largely theoretical ideals rather than everyday tools in oil and gas decision making.

The central contribution of this paper is to address a gap in petroleum project evaluation where managerial flexibility is often analyzed separately from managerial risk preferences. Although earlier studies have recognized the value of real options or the relevance of risk attitudes, few have provided a practical framework that integrates both within a single, operational methodology. Building on this need, the present work develops a fully implementable ROV–EUT approach that links flexible decision structures with risk-adjusted values derived from expected utility theory.

Unlike prior efforts (e.g., [13]), which discussed conceptual connections between utility functions and option valuation, the framework presented here offers an end-to-end workflow that combines influence-diagram modelling, probabilistic uncertainty characterization, and certainty-equivalent decision evaluation in a coherent and replicable process. A detailed case study further demonstrates how risk aversion reshapes expand–defer–abandon strategies and alters the economic value of managerial flexibility, revealing strategic patterns not previously documented.

By unifying flexibility valuation with explicit modelling of decision-maker preferences, this study advances the literature both methodologically and practically, providing petroleum decision makers with a realistic, transparent, and decision-consistent approach for evaluating investment opportunities under uncertainty.

The remainder of this paper is structured as follows. Sections 2 and 3 review the methodological foundations of real options and EUT, respectively. Section 4 outlines the valuation approach within the proposed methodology framework. Section 5 introduces the case study and presents and discusses the results of ROV under different risk preferences, highlighting how varying risk attitudes shape option values and optimal policies. Section 6 concludes with implications for petroleum engineers and geoscientists, and offers directions for further research.

## 2. ROV and Flexibility

ROV extends traditional NPV by explicitly valuing managerial flexibility under uncertainty [2, 3]. In ROV, an investment is viewed as a sequence of contingent decisions (delay, expand, abandon, switch inputs, etc.) rather than a one-off choice. The "value of flexibility" is the additional expected project value from having these options – essentially an option premium for the right (but not the obligation) to adapt later. For example, higher uncertainty in future prices or technical outcomes increases this premium, since upside is amplified while downside risk is contained by the option (unexercised options simply expire) [2, 5]. In oil-field exploration, this effect is dramatic; a field that appears unprofitable under static NPV can become profitable once the option to drill in stages or abandon is included [6, 18]. In short, real options add a time dimension and sequential decision-making to valuation: at each stage one compares the immediate NPV of investing now versus the continuation value of waiting, a process formalized by dynamic programming (DP) [19, 20]. If the "stopping value" (invest now) exceeds the "continuation value" (wait), it is optimal to invest; otherwise, the project is deferred. The foregone option to wait then becomes an opportunity cost that must be added to any NPV decision rule [2, 3].

Real options are used in many industries with large, uncertain projects – especially where staged or sequential choices arise. In oil and gas (and mining), ROV is now routine for exploration and field development. For example, companies bidding on oil licenses will pay more than static NPV because they can drill an appraisal well and then decide to develop or abandon [5, 6]. Empirical studies confirm this: Fedorov *et al*. (2020) model a marginal oil field where additional wells are drilled in two stages, and show that valuing the "option to wait and expand" via least-squares Monte Carlo adds significant upside [21]. Sarumpaet *et al*. (2025) similarly find that incorporating expand/abandon options via a binomial model raises an oil project's value by millions beyond NPV [18]. Such staged strategies let firms "buy time" – waiting for better oil-price or reservoir information – which is valuable under uncertainty.

Beyond hydrocarbons, ROV is applied wherever flexibility matters. Renewable energy and infrastructure projects (wind farms, solar plants, carbon-capture units, etc.) are evaluated with real options to capture volatile fuel prices and technology learning. In one study of a wave-energy venture, the embedded real-option value turned a negative NPV into a positive expected payoff by treating capacity expansion as an option [22]. Authors note that ROV is "already being applied frequently" in renewables and carbon-capture/storage projects [6]. Similarly, pharmaceutical R&D pipelines, tech startups, and manufacturing lines use real options: each new drug trial or product launch can be delayed or abandoned, and projects are often stage-gated [2]. Even flexible manufacturing (input/output switching) is valued via options [23]. In all these sectors, ROV systematically quantifies the extra payoff from strategic flexibility under uncertainty.

## 2.1. Methodologies for ROV

The practical implementation of real options requires formal methodologies that can capture uncertainty, evaluate contingent cash flows, and identify optimal decision rules at different stages of a project. Over the past three decades, a range of quantitative approaches have been developed to operationalize real-options valuation, each with distinct strengths, limitations, and areas of application [2, 19, 20]. At their core, these methods are grounded in DP, in which the value of a project at any point is determined by comparing the payoff from immediate investment with the continuation value of waiting or pursuing alternative actions. In petroleum and other capital-intensive industries, where projects unfold through sequential exploration, development, and production decisions, these methodologies provide structured tools for quantifying the value of flexibility. The most commonly applied approaches include decision-tree analysis, binomial or multinomial lattices, simulation-based methods such as least-squares Monte Carlo, and analytical formulations based on partial differential equations. The following items summarize these approaches, highlighting their underlying logic, advantages, and practical relevance to petroleum project evaluation.

- *Decision-tree analysis:* A common approach is to build a decision tree of sequential choices and chance events. Decisions like "invest now or wait" branch into uncertain outcomes. Each path ends in a cash-flow payoff (leaf), and one "rolls back" the tree by computing EVs to pick the best strategy. Real options appear as decision branches (e.g. the branch "abandon if price low" vs "drill more if price high") [1, 5]. This method explicitly models managerial flexibility: for example, a wave-energy model used decision-tree nodes for the compound option of continuing development or not, under a risk-neutral price process [22]. Decision trees are intuitive and accommodate many scenarios, but can become large with many stages and states.
- *Binomial/multinomial lattice (tree) models*: These models discretize the uncertainty into up/down moves [19]. The underlying project value (e.g. NPV or resource price) moves up or down each period. With risk-neutral probabilities, the option value is found by backward induction. American-style features[1] (expand, abandon at any time) are handled by checking at each node whether exercising yields more than waiting. For example, Sarumpaet *et al*. (2025) built a binomial tree for an offshore oil field; by embedding abandon and expansion options, they showed total value well above the base-case NPV [18]. Lattice models converge to Black–Scholes as steps increase, but remain manageable for one or two uncertainties [19, 20].
- *Least-squares Monte Carlo (LSMC):* Longstaff and Schwartz's (2001) simulation uses many random price/cash-flow paths to value American-style options [25]. At each decision date, regression estimates the continuation value (the value of waiting) for each simulated state. One then compares it to the immediate exercise payoff to decide optimally. This method handles multiple risk factors and complex payoffs. Fedorov *et al*. (2020) used LSMC to value the option of delayed drilling in an oil field; by simulating oil-price and reservoir outcomes, they captured the upside of waiting for information [21]. LSMC is especially useful when analytical or tree-based methods are infeasible (high dimensional, path-dependent cases).

[1] *American option:* A type of financial derivative that grants the holder the right, but not the obligation, to exercise the option at any time up to and including its expiration date, offering greater flexibility [24].

- *Analytical/PDE models*: In special cases, one can derive a closed-form or PDE solution (analogue of Black–Scholes). For a single underlying following GBM and a European-style exercise[2], the Black–Scholes formula can apply [26]. However, real investments rarely fit these assumptions (multiple uncertainties, American exercise, stage costs). PDE methods give exact answers for simple low-dimensional real options, but become intractable as uncertainties rise. They are most often used to benchmark or approximate cases where the timing or exercise is fixed.
- *DP and Backward Induction*: Conceptually, all the above methods perform DP; at each stage compare "stop vs continue." For example, one formally solves for the option by working backward from the last period [19, 20]. DP partitions the multi-stage problem into subproblems and ensures that at each decision point the optimal choice is made (invest now or wait). In practice, tree methods and LSMC implement DP numerically by comparing immediate exercise value to expected future value [24].

Each methodology has trade-offs. Decision trees and lattice models are transparent and easy to implement for a few variables, while LSMC tackles more complex or high-dimensional uncertainty. Analytical/PDE solutions are precise but restrictive in scope. In all cases, the goal is to quantify the value of flexibility – the option value that makes an otherwise marginal project worthwhile – and to guide the sequential investment strategy that realizes that value [2, 3].

## 2.2. Binomial Decision Tree for ROV

This section provides a concise explanation of the binomial decision tree approach for valuing real options. It outlines how the calibrated volatility parameter is used to construct a recombining binomial lattice to model the project's underlying value and how decision nodes are embedded to represent managerial flexibilities.

### *2.2.1. Binomial Decision Tree Approach*

The binomial decision tree is a widely used numerical method for valuing real options. It's a flexible and intuitive approach that can handle complex option features and path-dependent payoffs. The core idea is to model the evolution of the underlying asset's value (e.g., project value, commodity price, exchange rate) over time using a discrete-time, recombining binomial lattice. By carefully constructing the binomial lattice, incorporating decision nodes, and calibrating the volatility parameter, analysts can obtain a more accurate and realistic valuation of real options.

### *2.2.2. Constructing the Binomial Lattice*

The binomial lattice represents the possible paths that the underlying asset's value can take over the life of the option. It starts with the current value of the underlying asset at time zero and branches out at each time step into two possible values: an "up" state and a "down" state.

- *Time steps*: The time horizon of the option is divided into a number of discrete time steps (e.g., years, months). The more time steps, the more accurate the valuation, but also the more computationally intensive.
- *Up and down factors*: The magnitudes of the "up" and "down" movements are determined by the up factor (u) and the down factor (d), respectively. These factors are typically calculated based on the volatility of the underlying asset and the length of the time step. A common approach is to use the following formulas:

$$u = e^{(\sigma\sqrt{\Delta t})} \quad (1)$$

$$d = \frac{1}{u} = e^{(-\sigma\sqrt{\Delta t})} \quad (2)$$

Where $\sigma$ denotes the volatility of the underlying asset, and $\Delta t$ is the length of the time step.

[2] *European option:* A type of financial derivative that allows the holder to exercise the option only on its expiration date, generally leading to simpler valuation models [24].

*Risk-neutral probabilities***:** To value the option, we need to calculate the risk-neutral probabilities of the "up" and "down" movements. These probabilities are not the actual probabilities of the asset's price moving up or down, but rather the probabilities that would make investors indifferent between investing in the asset and investing in a risk-free bond. The risk-neutral probability of an "up" movement (p) is calculated as:

$$p = \frac{e^{r\Delta t} - d}{u - d} \approx \frac{(1 + r\Delta t) - d}{u - d} \quad (3)$$

where *r* is the risk-free interest rate.

A schematic representation of the two-period binomial tree is provided in Fig. (**1**). The tree models the stochastic evolution of the project's value (*V*) over time using the up and down movements and the specified probabilities.

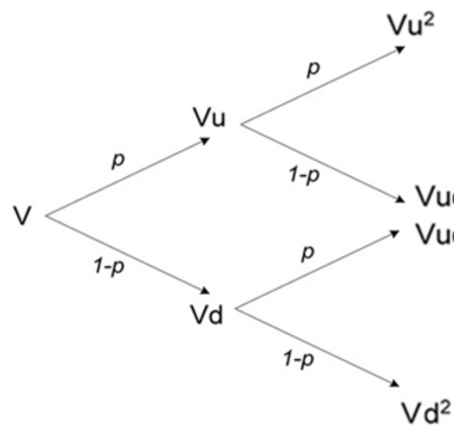


**Figure 1:** Schematic representation of a binomial decision tree.

### *2.2.3. Incorporating Decision Nodes*

Real options involve managerial flexibility, meaning that management can make decisions at certain points in time that affect the value of the project. These decision points are represented as decision nodes in the binomial lattice.

- *Exercise dates*: Decision nodes are placed at the dates when management has the option to take a specific action, such as abandoning the project, expanding production, or switching to a different technology.
- *Decision rule***:** At each decision node, the value of the project is calculated under each possible decision. Management will choose the decision that maximizes the project's value:

$$V_{decision} = \max(V_{continue}, V_{exercise}) \quad (4)$$

where $V_{continue}$ is the value of continuing the project, and $V_{exercise}$ is the value of exercising the option (e.g., abandoning, expanding). For example, if the option is to abandon the project, management will compare the value of continuing the project with the salvage value of abandoning it and choose the higher value.

### *2.2.4. Backward Induction*

Once the binomial lattice is constructed and the decision nodes are incorporated, the project's expected NPV is calculated using backward induction. This involves starting at the end of the lattice (the expiration date of the option) and working backward to the present.

- *Terminal values***:** At the final time step, the value of the option is determined by the payoff of the option at expiration. For example, if the option is to expand production, the payoff at expiration will be the incremental profit from expanding production.

- *Rolling backwards***:** At each node in the lattice, the value of the option is calculated as the discounted EV of the option in the next time step, using the risk-neutral probabilities:

$$V_t = e^{-r \cdot \Delta t} \cdot \left[p \cdot V_{t+1,up} + (1-p) \cdot V_{t+1,down}\right] \quad (5)$$

At decision nodes, the value of the option is the maximum of the value of exercising the option and the value of continuing the project as outlined in Eq. 4.

- *Option value:* The value at the initial node (time zero) represents the expected NPV of the entire project, incorporating the value of the strategic flexibility. The value of the real option itself is calculated as the difference between the expected NPV with the flexibility (real options) incorporated and the standard NPV of the project without the flexibility.

#### *2.2.5. Calibrated Volatility*

The volatility parameter ($\sigma$) is a critical input to the binomial decision tree model. It reflects the uncertainty surrounding the project's future value. In real options analysis, it's often necessary to calibrate the volatility parameter to reflect the specific characteristics of the project and the market in which it operates. This can be done by using either historical data on similar projects or assets, market data on traded options on similar assets, or MCS to estimate the volatility of the project's cash flows.

## 3. Modeling Risk Attitudes in Petroleum Decision-Making

Petroleum projects are characterized by deep uncertainty, high capital intensity, and often irreversible commitments. Decisions such as whether to drill an exploration well, proceed with field development, or abandon a marginal asset involve substantial financial stakes and uncertain outcomes. Traditionally, many organizations have relied on expected monetary value (EMV) as the primary decision criterion. While EMV provides a simple benchmark, it implicitly assumes that DMs are risk-neutral, valuing all monetary outcomes in proportion to their magnitude. In practice, DMs often deviate from risk neutrality due to limited budgets, corporate risk policies, or psychological aversion to downside outcomes. For example, two exploration prospects may have identical EMVs, but one offers a small probability of a major discovery while the other yields a high probability of a modest return. Risk-neutral evaluation would treat these equally, yet managers may prefer the latter due to its reduced downside exposure. This highlights the importance of formally modeling risk attitudes in petroleum economics and decision analysis [27, 28].

### 3.1. Alternative Metrics for Risk Preferences

Several practical methods have been developed to account for risk attitudes:

- *Mean–variance analysis* compares investment alternatives by weighing their expected return against the variability of possible outcomes, typically measured using variance or standard deviation [29]. While this approach has been highly influential in finance, it is only reliable under restrictive conditions—such as when returns are normally distributed—because variance alone cannot capture more complex aspects of risk. In petroleum projects, where outcomes are often skewed and subject to extreme events, the mean–variance framework provides at best an incomplete and potentially misleading representation of risk.
- *VaR* measures the worst expected loss at a given confidence level [30]. While widely used, VaR ignores losses beyond the threshold and may violate consistency properties desirable for coherent risk measures.
- *PoS* emphasizes the likelihood of exceeding a technical or economic threshold. While intuitive for exploration drilling, it neglects payoff magnitude and may favor high-probability, low-value projects.

Although these approaches provide practical insight, they do not ensure consistent decision-making across complex project portfolios and cannot capture the full spectrum of risk preferences. A common approach in petroleum economics is to incorporate risk aversion via risk-adjusted discount rates in NPV calculations. However, decision analysis has shown that this conflates risk attitude and time preference, which are fundamentally distinct. Treating these together in a single discount rate may obscure true decision preferences and is generally discouraged.

### 3.2. EUT: Foundations

Prompted by Bernoulli's (1738) work on the St. Petersburg paradox [31], the utility concept captures the subjective value of monetary outcomes, recognizing that the same amount of money may be worth more to a poorer person than a wealthier one [32]. Von Neumann and Morgenstern (1944) formalized this idea as EUT [10], a normative model for rational decision-making under uncertainty adopted across decision theory, economics, and game theory [33, 34]. Metrics inconsistent with EUT can lead to decisions that do not reflect the DM's actual risk preference.

EUT is built on four axioms ensuring rational choice:

1. Completeness: All alternatives can be ranked or declared indifferent.
2. Transitivity: If option A is preferred to B, and B to C, then A is preferred to C.
3. Independence: Preferences between lotteries remain unchanged when identical outcomes are added (If A is preferred to B, then the preference between lotteries involving A and B remains unchanged when combined with a third outcome).
4. Continuity: If A is preferred to B and B to C, a probabilistic mixture of A and C exists that is equivalent to B.

Compliance with these axioms ensures that a utility function exists and that DMs act consistently to maximize expected utility (EU) rather than EMV alone. A rational DM then selects the option that maximizes EU:

$$EU = \sum_i p_i U(x_i) \tag{6}$$

Where $p_i$ is outcome probability and $U(x_i)$ is the utility of outcome $x_i$.

### 3.3. Utility Functions and Certainty Equivalents (CE)

A utility function is a mathematical representation of a DM's preferences, capturing sensitivity to changes in wealth relative to a reference point and attitudes toward uncertainty. The CE of a risky prospect is the guaranteed payoff a DM considers equivalent to the uncertain outcome. Formally, the CE of a deal can be calculated by first calculating the EU of the deal and then calculating the equivalent monetary value that has the same utility as the EU:

$$CE = u^{-1}(EU) - w_0 \tag{7}$$

where $u^{-1}(\cdot)$ is the inverse utility function and $w_0$ is the initial wealth. Using CE allows risk-averse or risk-seeking preferences to be incorporated directly into monetary terms, ensuring rational, transparent, and traceable decisions. For risk-neutral DMs, with linear utility functions, CE coincides with expected value (EV), while for risk-averse or risk-seeking DMs, EV alone is inconsistent.

Utility functions must be non-decreasing in wealth and can take different forms. The shape of an individual's utility function provides valuable insights into their attitude towards risk:

#### *3.3.1. Linear Utility: Risk Neutrality*

A linear utility function implies that an individual is risk-neutral. This means they are indifferent between receiving a certain amount of money and participating in a gamble with the same EV. The utility function takes the form:

$$U(w) = aw \tag{8}$$

where *U(w)* is the utility derived from wealth *w*, and *a* is a positive constant representing the marginal utility of wealth.

### *3.3.2. Concave Utility: Risk Aversion*

A concave utility function implies that an individual is risk-averse. This means they prefer receiving a certain amount of money to participating in a gamble with the same EV. The utility function exhibits diminishing marginal utility of wealth. A common example is:

$$U(w) = \sqrt{w} \tag{9}$$

### *3.3.3. Convex Utility: Risk Seeking*

A convex utility function implies that an individual is risk-seeking. This means they prefer participating in a gamble to receiving a certain amount of money equal to the gamble's EV. The utility function exhibits increasing marginal utility of wealth. An example is:

$$U(w) = w^2 \tag{10}$$

The relationship between the CE and the EV of a gamble is a key indicator of these risk preferences. Understanding these concepts is crucial for analyzing decision-making under uncertainty in various fields, including economics, finance, and insurance. The characteristics of different utility function and their corresponding risk attitudes are summarized in Table **1**.

**Table 1: Characteristics of different risk attitudes.**

| Type of Utility Function | Risk Attitude | Marginal Utility | Comparison of CE Against EV |
|---|---|---|---|
| Linear | Risk-neutrality | **Constant Marginal Utility:** The marginal utility of wealth (the change in utility from a small increase in wealth) is constant. This means that each additional unit of wealth provides the same amount of additional satisfaction, regardless of the individual's current wealth level. | **CE equals EV:** For a risk-neutral individual, the CE of a gamble is equal to its EV. The CE is the amount of certain wealth that would provide the same level of utility as the gamble. Since the individual is indifferent between the gamble and its EV, they are willing to accept the EV as a certain payment. |
| Concave | Risk-aversion | **Diminishing Marginal Utility:** The marginal utility of wealth decreases as wealth increases. This means that each additional unit of wealth provides less additional satisfaction than the previous unit. The individual values the first few dollars more than the last few. | **CE is less than EV:** For a risk-averse individual, the CE of a gamble is less than its EV. This reflects the fact that they are willing to accept a smaller certain payment to avoid the risk associated with the gamble. The difference between the EV and the CE is called the risk premium. |
| Convex | Risk-seeking | **Increasing Marginal Utility:** The marginal utility of wealth increases as wealth increases. This means that each additional unit of wealth provides more additional satisfaction than the previous unit. | **CE is greater than EV:** For a risk-seeking individual, the CE of a gamble is greater than its EV. This reflects the fact that they are willing to accept a lower EV to have the chance of a higher payoff. |

## 3.4. Delta Property and Exponential Utility

A delta person is a DM whose CE increases by the same amount if all possible payoffs increase by that amount [34]. For delta persons, CE is independent of initial wealth, allowing $w_0$ to be set arbitrarily—commonly zero for simplicity. Only two utility functions satisfy this property: linear and exponential utility functions. The exponential utility function provides a flexible and intuitive framework for modeling risk preferences (Eq. 11):

$$u(x) = \left(1 - e^{-\frac{x}{\rho}}\right)\rho \tag{11}$$

where $x$ is the payoff and $\rho$ represents risk tolerance. The parameter $\rho$ effectively captures the degree of risk aversion or risk seeking, allowing for a nuanced understanding of how individuals and firms make decisions under uncertainty. Positive $\rho$ indicates risk aversion because with a positive $\rho$, the utility function is concave. A negative value of $\rho$, however, indicates risk seeking since with a negative $\rho$, the utility function is convex. As the absolute value of $\rho$ approaches infinity, the utility function approaches a linear utility function. This represents risk neutrality, where the DM is indifferent between a certain outcome and a risky outcome with the same EV.

An extremely small positive $\rho$ models extreme risk aversion. In this scenario, a company might accept a lower guaranteed payment to avoid the risk of failure, even if the EV of the risky option is significantly higher. This behavior is often observed in situations where the consequences of failure are catastrophic. Conversely, an extremely small negative $\rho$ reflects extreme risk seeking. In this scenario, firms pursue highly uncertain projects with potentially high payoffs, even if the probability of success is very low. This behavior is often observed in industries with winner-take-all dynamics, such as venture capital or biotechnology.

In petroleum applications, these concepts allow analysts to quantify preferences consistently, compare projects with different risk profiles, and make decisions aligned with corporate risk strategies.

Determining an appropriate value for the risk-tolerance parameter is essential for aligning the expected-utility formulation with a specific organization's decision posture. A brief set of practitioner-oriented guidelines and elicitation methods for estimating ρ is provided in Appendix **A.1**, enabling the framework to be aligned with real corporate decision-making behaviour.

A simple illustrative example is provided in Appendix **A.2** to demonstrate how expected utility and certainty equivalents differ under various risk attitudes. Here, our focus remains on the conceptual role of EUT in shaping investment preferences.

## 4. Proposed Methodology for ROV in Oil Production Projects

This section outlines a systematic, generalizable methodology for valuing investment projects embedded with managerial flexibility (real options) while explicitly incorporating the DM's risk preferences. The procedure moves beyond standard risk-neutral valuation by employing EUT, thus providing a CE value that is consistent with a rational DM's subjective attitude towards risk. The methodology extends the foundational work of [5] on binomial decision trees for real options and incorporates the comparative valuation framework under alternative risk attitudes advanced by [35]. The following step-by-step framework, summarized in Fig. (**2**), can be applied to value flexibility in various capital-intensive and uncertain environments.

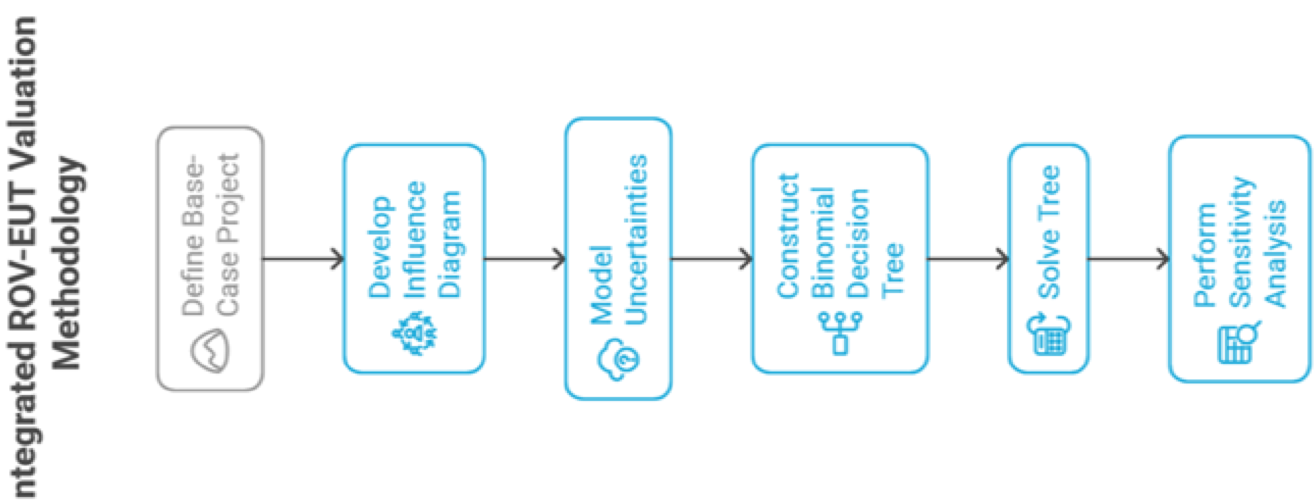


**Figure 2:** Flowchart of the proposed methodology for ROV under risk preferences.

### 4.1. Project Definition and Option Specification

The first step involves a precise definition of the investment project under a passive management scenario. This includes detailing all relevant technical and economic parameters, projected cash flows, and the project's life. Subsequently, the strategic managerial flexibilities (real options) must be explicitly specified. This includes defining their type (e.g., option to abandon, expand, defer, or switch), the timing at which they can be exercised, the costs associated with exercise, and the payoffs they generate.

### 4.2. Structural Model Design via Influence Diagram

Before quantitative analysis, the problem must be structured conceptually. An influence diagram is constructed to visually map the relationships between decisions, uncertainties, and outcomes. This diagram includes:

- *Decision nodes*: representing the points where managerial choices (e.g., exercising an option) are made.
- *Chance nodes*: representing the key uncertain variables that drive project value (e.g., market prices, costs, demand).
- *Value node*: representing the objective, typically the NPV of the project.

In addition, of critical importance are information arcs from chance nodes to decision nodes, which signify that decisions are made with knowledge of the resolved uncertainties up to that point.

This diagram serves as the logical blueprint for the subsequent quantitative model, ensuring all dependencies are correctly represented.

### 4.3. Stochastic Modeling and Volatility Estimation

The uncertainties identified in the influence diagram are modeled as stochastic processes, with Geometric Brownian Motion (GBM) being a common choice for market-price-related risks. Other price models can also be used depending on the specific characteristics of the risks being modeled. The selection of appropriate stochastic process is crucial for accurately capturing the dynamics of the underlying uncertainties. The project’s overall uncertainty is summarized through a volatility estimate derived via Monte Carlo simulation. The full simulation procedure is documented in Appendix **A.3**, while this section concentrates on how the resulting volatility parameter feeds into the real options framework.

### 4.4. Binomial Decision Tree Construction

The binomial decision tree provides the computational structure for valuation. The tree is constructed by calculating the up ($u$) and down ($d$) factors that define the branches at each time step ($\Delta t$). The risk-neutral probabilities ($p$) for these branches are calculated using the risk-free rate (see Eqs. 1-3). The tree is then populated by translating the structure of the influence diagram. Decision nodes are inserted at the contractually specified times, branching out into the available strategic alternatives.

### 4.5. Valuation Framework: Risk-Neutral and Risk-Averse Analysis

The tree is solved using backward DP.

*Risk-Neutral Valuation:* This approach values the project as if the DM is risk-neutral. At each node, the value is the EV of future outcomes, discounted at the risk-free rate.

At a chance node, the value $V_t$ is calculated using Eq. 5. At a decision node, the value is the maximum of the values of the subsequent branches:

$$V_{decision} = \max\left(V_{Option\ 1}, V_{Option\ 2}, V_{Option\ N}\right) \quad (12)$$

The result is a risk-neutral value of strategic flexibility.

*Risk-Averse Valuation via EUT*: This approach incorporates risk preference. An exponential utility function (Eq. 11) is used for its mathematical properties and simplicity. The backward induction process operates on utilities:

- At terminal nodes, the utility of the final cash flow is computed.
- At a chance node, the EU is calculated and converted to its CE, which is the sure amount of money with the same utility:

$$EU = p \cdot U(V^u) + (1-p) \cdot U(V^d) \tag{13}$$

$$CE = U^{-1}(EU) = -\rho \, \mathbf{ln}(1 - \rho . EU) \tag{14}$$

This CE is then discounted at the risk-free rate.

- At a decision node, the CE of each alternative is calculated, and the path with the highest CE is selected.

The final result is the CE value of the project for a DM with a specified risk tolerance $\rho$.

### 4.6. Sensitivity Analysis

A sensitivity analysis is conducted on the risk tolerance parameter $\rho$ to explore its impact on the project's valuation and the optimal decision policy. The model is solved for a spectrum of $\rho$ values, from very low (extreme risk aversion) to very high (approaching risk neutrality). This analysis reveals how the value of different real options and the overall strategic recommendation evolve with the DM's degree of risk aversion.

## 5. Results and Discussion

This study utilizes a modified version of a pedagogical oil-field investment case to demonstrate the application of ROV techniques under different risk preferences. A simplified oil production project, inspired by the [5], known as BDH model, was adopted as the case study. Following the procedure outlined in the previous section, the project's embedded flexibilities are valued using a binomial decision tree, the dynamics of which are calibrated from an MCS of the project's underlying uncertainties. Valuation is performed using two complementary paradigms: (1) risk-neutral valuation, which assumes investors are indifferent to risk and discounts expected payoffs at the risk-free rate, and (2) EUT, which explicitly models a risk-averse DM whose choices maximize EU. A sensitivity analysis on the DM's level of risk aversion is conducted to assess its impact on optimal exercise policies and option value. Decision modeling was performed using the professional DPL software to develop inference diagrams, create and solve decision trees, and conduct sensitivity analysis.

### 5.1. Case Study Definition and Deterministic Base Case

To demonstrate the practical application and value of the integrated ROV-EUT methodology, we employ a modified version of the canonical oilfield investment case presented by [5], henceforth referred to as the BDH problem. This case provides a transparent and tractable numerical example, ideal for elucidating the impact of risk aversion on strategic optionality. For enhanced clarity in presenting the decision tree outcomes, we have simplified the original model by condensing the operational timeline.

The project entails the development of an oil field with initial reserves of 90 million barrels, to be produced over a four-year period. Production initiates at 9 million barrels per year, declining at an annual rate of 15%. The economic framework is characterized by the following parameters:

- The initial oil price is $25 per barrel, escalating at a risk-adjusted rate of 3% per year.
- Variable operating costs commence at $10 per barrel, increasing at 2% annually.

- Fixed operating costs are $5 million per year.
- The project is evaluated using a risk-free rate of 5%.
- The developer holds a 75% ownership.

The passive value of this project is subject to significant uncertainty, driven by two primary, non-diversifiable market risks: the volatile market price of oil and fluctuating variable operating costs.

The strategic focus of this analysis is on two distinct real options granted to the developer at the end of the second year of production:

- *The Option to Divest (Abandon):* The right to sell the entire project asset for a guaranteed payment of $100 million.
- *The Option to Buy Out (Expand):* The right to acquire the remaining 25% ownership for a cost of $40 million, thereby increasing the developer's project share to 100% for all subsequent cash flows.

The following section provides a quantitative valuation of these alternatives under both risk-neutral and risk-averse frameworks, demonstrating how the optimal strategy is profoundly shaped by the DM's subjective risk preferences.

### 5.2. Framing Decision Problem Using Inference Diagram

The influence diagram for the modified BDH problem is shown in Fig. (**3**). It structurally captures the key elements of the real options decision process across four annual time steps ($T_1$ to $T_4$). The diagram includes chance nodes representing the stochastic evolution of the project value ($PV_0$) evolving over time with volatility *Vol*, influenced by the annual cash flow payout rate ($d_1$ to $d_4$), as will be described later in this section. A decision node is embedded at $T_2$, where the manager chooses between three alternatives: to continue, to divest (receiving a fixed sum), or to buy out the remaining stake (increasing future cash flow entitlement). The diagram also incorporates the risk-free rate *rf* and risk-neutral probabilities (*p*, *1-p*) governing upward *u* and downward *d* movements in project value, ensuring the valuation remains arbitrage-free. This representation clarifies the dependencies between market uncertainties, intermediate decisions, and final value, providing a transparent roadmap for constructing the binomial decision tree and applying backward induction under both risk-neutral and risk-averse frameworks.

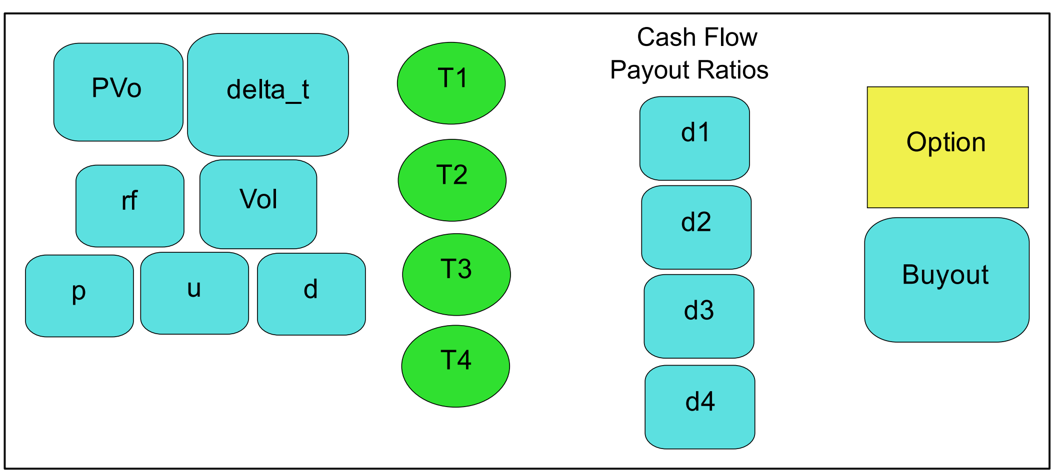


**Figure 3:** Influence diagram for the modified BDH decision problem.

A deterministic DCF model is first constructed to establish the passive NPV of the project, assuming no managerial flexibility throughout its operating life. This static NPV serves as a benchmark against which the value of active strategic management is compared.

To evaluate the project, we need to project the cash flows for each year. The cash flow for each year can be calculated as follows:

$$Cash\ Flow = (Oil\ Price - Variable\ Operating\ Cost) * Production\ Volume - Fixed\ Cost \quad (15)$$

Fig. (**4**) expands the base case cash flow projections over 4 years for the BDH problem. Drawing on the equilibrium approach for this ROV problem, the risk-neutral drift for oil prices is set to 0% per year. This presumes that long-term future prices or forward contracts do not indicate significant predictable changes. In contrast, operating costs are assumed to be non-tradable and uncorrelated with the market portfolio; therefore, no risk adjustment is required. Consequently, the risk-neutral drift for operating costs is set equal to their true drift of 2% per year.

| Year | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Reserve | 90 | | | | |
| Remianing Reserves | | 90 | 81.0 | 73.4 | 66.8 |
| Initial Production Level | 9 | | | | |
| Yearly Decline Rate | 15% | | | | |
| Production Level | | 9 | 7.7 | 6.5 | 5.5 |
| Variable Operating Cost at Start Year | 10 | | | | |
| Cost Growth Rate | 0.02 | | | | |
| Variable Op Cost Rate | 10 | 10.2 | 10.4 | 10.6 | 10.8 |
| Oil Price at Start Year | 25 | | | | |
| Price Growth Rate | 0 | | | | |
| Oil Price | 25 | 25.0 | 25.0 | 25.0 | 25.0 |
| Revenues | | 225.0 | 191.3 | 162.6 | 138.2 |
| Yearly Fixed Cost | 5 | | | | |
| Production Cost | | 96.8 | 84.6 | 74.0 | 64.8 |
| Cash Flow | | 128.2 | 106.7 | 88.6 | 73.4 |
| Profit Sharing Percentage | 0.25 | | | | |
| Profit Sharing | | 32.1 | 26.7 | 22.1 | 18.3 |
| Net Cash Flows | | 96.2 | 80.0 | 66.4 | 55.0 |
| Risk-Adjusted Discount Rate | 0.05 | | | | |
| PV of Cash Flows | 266.8 | 280.1 | 193.1 | 118.8 | 55.0 |
| Cash Flow Payout Rate | | 0.343 | 0.414 | 0.559 | 1.000 |

**Figure 4:** Base case cash flow projections for the modified BDH problem.

The cash flow payout rate is defined as the ratio of the cash flow received ($C_i$) to the project value ($V_i$) at a given time period, expressed as $\frac{Ci}{Vi}$. This rate represents the fraction of the project's total value that is distributed as a cash payout at the end of each period. A key assumption is that this payout rate remains constant over time, meaning that cash flows fluctuate in proportion to changes in the project's residual value. These cash flows are thus directly tied to the stochastic evolution of the project value within the binomial tree model. This approach enhances modeling flexibility for real options analysis, as it dynamically links payouts to underlying project uncertainty [5].

### 5.3. Conducting MCS of Project Values Using GBM Approximation and Estimating Project Volatility

This step involves simulating the project's value over time using MCS and GBM. The goal is to estimate the project's volatility, which is a crucial input for constructing the binomial tree. GBM is a stochastic process often used to model the evolution of asset prices over time. It assumes that the percentage changes in the project's value are normally distributed. The GBM equation is:

$$dV = \mu V dt + \sigma V dz \quad (16)$$

where:

- *dV* is the change in project value.
- *V* is the current project value.
- *μ* is the drift rate (expected rate of return).
- *dt* is the change in time.
- *σ* is the volatility of the project value.
- *dz* is a random increment drawn from a standard normal distribution.

First, the key factors that influence the project's value should be identified. These drivers include oil price and variable operating cost for the current project. Next, a large number of random scenarios (e.g., 10,000) is generated by sampling from the probability distributions of the project value drivers. For each scenario, the project's value at each time step is calculated using the GBM equation. Finally, the project values for each scenario are stored at each time step.

After running the MCS, the project's volatility (*σ*) is estimated from the simulated project values. This can be done by calculating the standard deviation of the logarithmic returns of the project value (Eqs. A9 and A10):

$$\mathrm{z} = \ln\left(\frac{V_1}{V_0}\right) \Rightarrow s = std(z) = 0.318 \ \Rightarrow \sigma = s\sqrt{\Delta t} = 0.318 \tag{17}$$

The binomial tree is a discrete-time model that represents the possible paths of the project's value over time. The parameters of the binomial tree are up and down movements (*u* and *d*), and risk-neutral probability (*p*). These parameters can be calculated using the following formulas (Eqs. 2 and 3):

$$u = e^{\sigma\sqrt{\Delta t}} = 1.37,\ d = \frac{1}{u} = 0.73 \tag{18}$$

$$p = \frac{1 + r\Delta t - d}{u - d} = 0.498 \tag{19}$$

### 5.4. Constructing binomial decision tree

This step involves constructing the decision tree, both with and without considering managerial options. The tree without options represents the project's value under different scenarios without any flexibility to make decisions along the way. The tree with options incorporates managerial flexibility by allowing for decisions to be made at different points in time. The binomial decision tree used for valuation of real options is illustrated in Fig. (**5**). Its structure is derived directly from the logical framework of the influence diagram presented in Fig. (**3**), translating conceptual nodes into computational ones: chance nodes model the stochastic evolution of project value, while the decision node at *t*=2 encapsulates the strategic choices to continue, divest, or buy out the remaining stake.

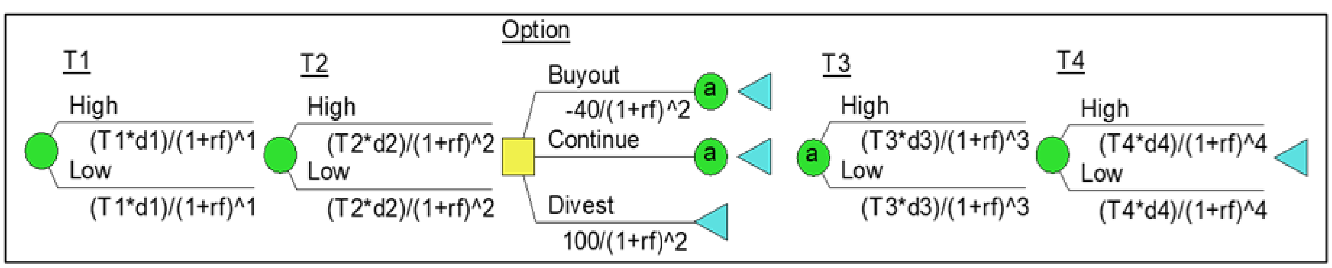


**Figure 5:** The binomial tree structure with options.

Each *circle* represents a chance node, corresponding to the resolution of market uncertainties—specifically, the oil price and operating cost—resulting in either an upward ('High') or downward ('Low') movement in the project value ($T_t$) at each annual time step. The letter 'a' inside certain circles denotes algorithmic state labels used by DPL to uniquely identify each node during the backward induction process.

Cash flows are incorporated at each period via the cash flow payout rate ($d_t$). The value $\frac{T_t \cdot d_t}{(1+rf)^t}$ represents the DCF received at the end of period *t*, where *rf* is the risk-free rate.

A key feature of the tree is the decision node (represented by a square) at *t*=2. Here, the DM must choose between three mutually exclusive strategies:

- *Buy Out*: Paying $40 million (discounted to present value as $-\frac{40}{(1+rf)^2}$) to acquire the remaining 25% ownership, thereby increasing ownership to 100% for all subsequent cash flows.
- *Continue*: Maintaining the current 75% stake without any additional investment or divestment.
- *Divest*: Selling the entire project for a guaranteed payment of $100 million (discounted as $\frac{100}{(1+rf)^2}$).

Triangles at the end of branches (e.g., at *t*=4) represent terminal nodes, indicating the conclusion of the project's life and the cumulative value of all cash flows received along that specific path.

## 5.5. Solving Decision Tree and Valuating Options

The tree is solved using DP via backward induction. Starting from the terminal nodes, the algorithm calculates the EV at each chance node under risk-neutral probabilities, and at the decision node, it selects the alternative that maximizes the project value. This process yields the risk-neutral value of the project including flexibility, and—when integrated with the exponential utility function—the Certain Equivalent value for a risk-averse DM. The explicit structure of the tree allows for clear interpretation of the optimal policy: under which market scenarios (i.e., which 'High' or 'Low' paths) it is advantageous to divest, buy out, or simply continue, providing actionable insight into the strategic value of the embedded options.

The solved binomial decision tree, presented in Fig. (**6**), provides a complete valuation of the project under a risk-neutral framework. The computed NPV of the project, incorporating the value of strategic flexibility, is $280.9 million. This value, located at the root node of the decision tree, represents the EV of the project when managed optimally through the exercise of the embedded real options.

The tree structure elucidates the state-dependent valuation and the decision logic leading to this result. The values within the chance nodes represent the project's NPV conditional on the state of the world at that time period. The branches emanating from these nodes represent the possible stochastic evolution of project value, with the associated risk-neutral probabilities (e.g., 50%) indicating the likelihood of each upward or downward movement. The intersection operator on these branches signifies the partition of the parent chance node, with the percentage value quantifying the conditional probability of transitioning to that specific child node.

A critical output of the analysis is the optimal policy chart, which summarizes the strategic prescription for the decision point at the end of Year 2. The results indicate that the optimal course of action is highly state-dependent:

- The option to *Continue* with the current working interest is optimal in 50% of the simulated future scenarios.
- The option to *Divest* the project for a guaranteed payment is the preferred strategy in 25% of the outcomes.
- The option to *Buy Out* the remaining stake is value-maximizing in the remaining 25% of the potential futures.

This distribution of optimal strategies offers profound strategic insight. The high frequency of the "Continue" strategy suggests that in a majority of future states, the project's prospects are sufficiently strong that maintaining the status quo is more valuable than a guaranteed exit or further capital commitment. The selection of the

"Divest" option in one quarter of the scenarios demonstrates the value of the abandonment option in mitigating losses under less favorable market conditions. Conversely, the "Buy Out" strategy is reserved for the most optimistic states, where the potential upside justifies the additional investment to gain full ownership. This quantitative breakdown moves beyond a single valuation metric and provides a dynamic, state-contingent roadmap for strategic decision-making, underscoring the critical value of managerial flexibility in this investment.

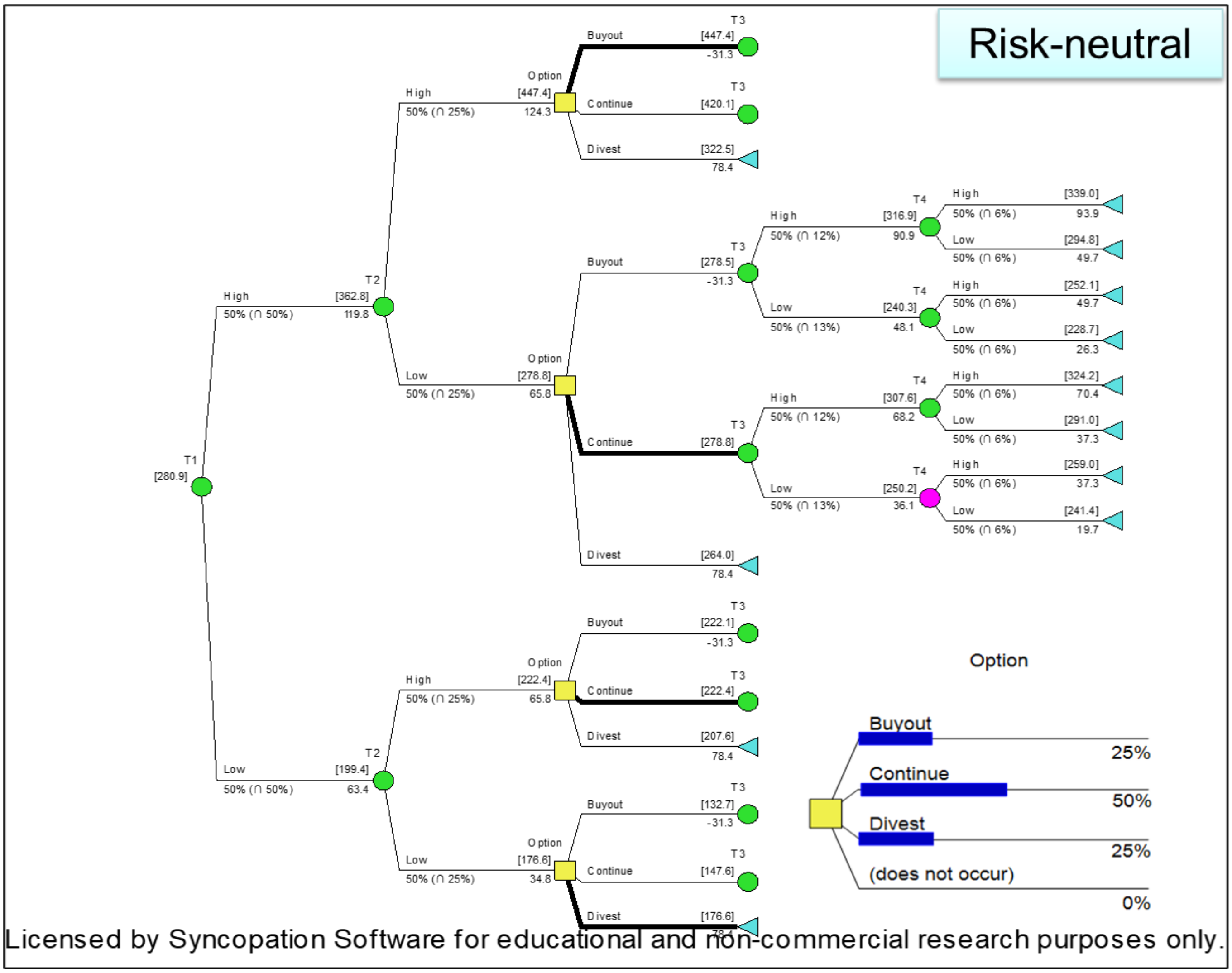


**Figure 6:** Solved decision tree for ROV under risk-neutrality for the modified BDH problem.

The decision tree was then solved under a risk-averse framework using an exponential utility function with a risk tolerance parameter ($\rho$) of \$20 million. The results, presented in Fig. (**7**), demonstrate a profound shift in both project valuation and optimal strategy compared to the risk-neutral case.

The CE value of the project for this risk-averse DM is \$200.2 million. This valuation is significantly lower than the risk-neutral NPV of \$280.9 million, reflecting the substantial risk premium the DM requires to compensate for the project's inherent uncertainty.

The optimal policy undergoes a dramatic transformation under risk aversion:

- The *Divest* option becomes the optimal strategy in 75% of future scenarios.
- The *Buy Out* option is selected in the remaining 25% of cases.
- The option to *Continue* is never optimal.

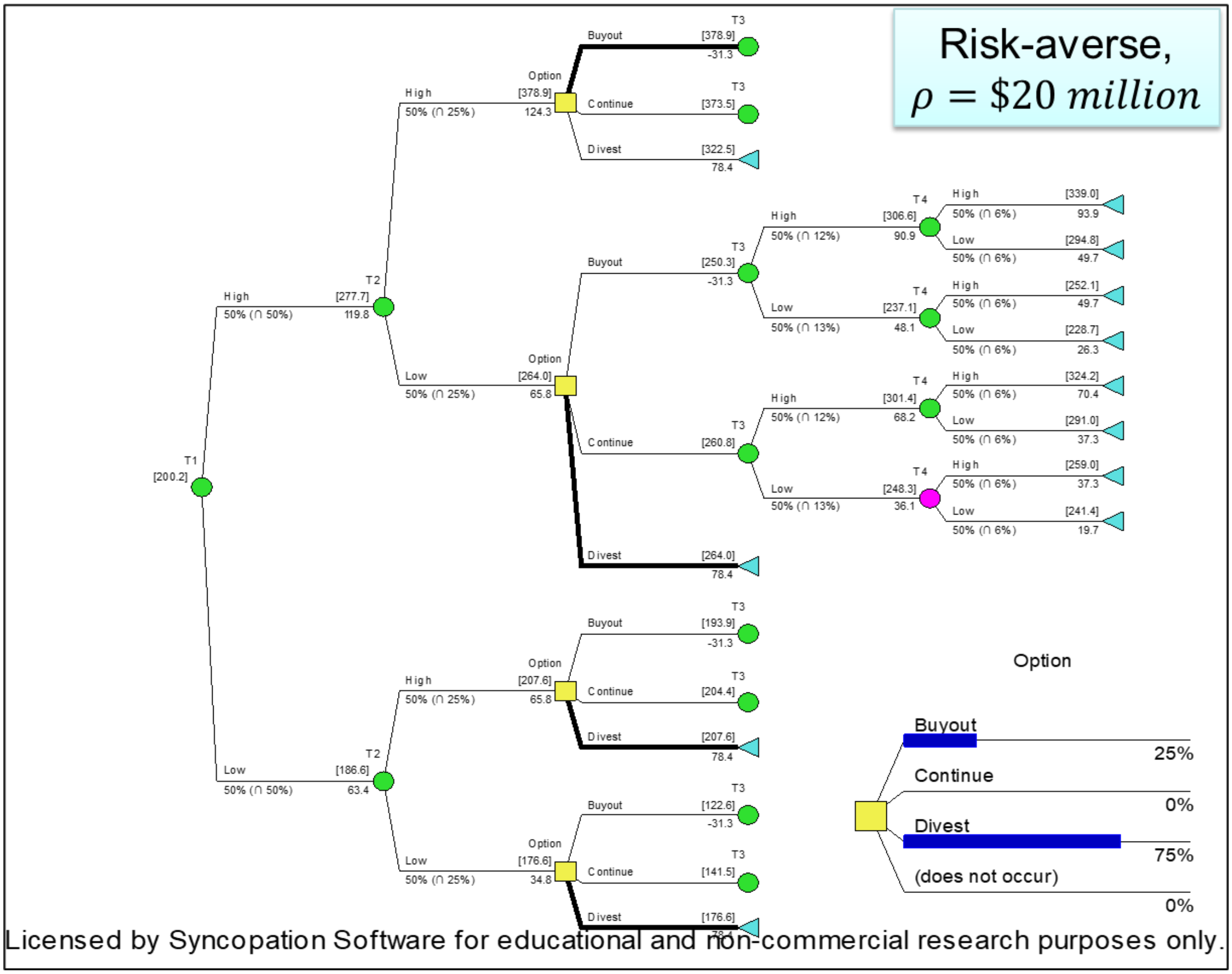


**Figure 7:** Solved decision tree for ROV under risk-aversion for the modified BDH problem.

The contrast between the risk-neutral and risk-averse results is striking and carries critical implications for strategic decision-making.

1. *Shift in Optimal Policy*: The risk-neutral DM, focused solely on EV, finds it optimal to Continue in 50% of cases. In contrast, the risk-averse DM never chooses to continue. This is because the "Continue" strategy exposes the DM to further unresolved uncertainty without any guaranteed payoff. The risk-averse party shows a strong preference for the certainty provided by the Divest option, exercising it in 75% of scenarios to lock in a sure payoff and eliminate downside risk.
2. *Nature of the Buy Out Option*: While the Buy Out option is selected in 25% of cases under both valuations, its nature changes. For the risk-neutral party, it is a leveraged bet on significant upside. For the risk-averse party, it is likely only selected in the most exceptionally favorable scenarios where the potential payoff is so high that it outweighs the acute discomfort of taking on more risk.
3. *Strategic Implications*: This analysis demonstrates that the value of flexibility is not absolute but is intrinsically tied to the risk preferences of the owner. A company with a high-risk tolerance (low risk aversion) would see greater value in maintaining and expanding its position. Conversely, a more risk-averse company would derive greater value from the same project by utilizing the abandonment option much more frequently, effectively using the real option as an insurance policy. This finding underscores that investment decisions and optimal strategy cannot be prescribed based on a single market-derived value; they must be contextualized within the specific risk appetite of the decision-making entity.

In conclusion, incorporating risk aversion through EUT does not merely adjust the project's value downward; it fundamentally alters the prescribed optimal management strategy, favoring decisive actions that limit risk exposure over passive continuation.

### 5.6. Sensitivity Analysis

Fig. (**8**) presents the results of a sensitivity analysis on the risk tolerance parameter ($\rho$), illustrating its profound impact on both the CE value of the project and the optimal decision policy. The plot charts the CE value (in $ Million) against a spectrum of risk tolerance values, effectively mapping the transition from extreme risk aversion to near risk-neutrality.

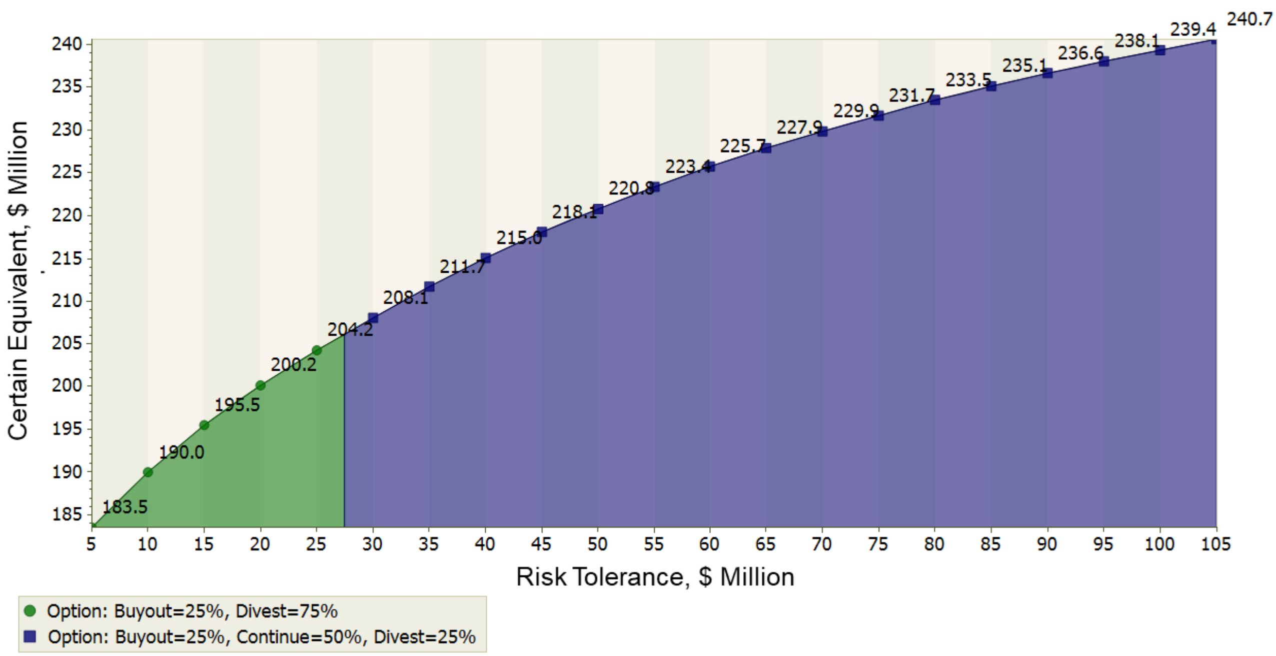


**Figure 8:** Sensitivity analysis of CE values of a risk-averse DM to his/ her risk tolerances for the modified BDH problem.

Two distinct optimal policy regions are identified and shaded on the graph:

- *Region 1 (Green Shading)*: This region, which prevails for lower risk tolerance values ($\rho$ below approximately $30 million), is characterized by the optimal policy {Buyout=25%, Divest=75%}. In this state, the option to Continue is never optimal.
- *Region 2 (Purple Shading)*: For higher risk tolerance values ($\rho$ above approximately $30 million), the optimal policy shifts to {Buyout=25%, Continue=50%, Divest=25%}.

The sensitivity analysis reveals several critical insights:

- *Valuation sensitivity:* The CE value increases monotonically with risk tolerance. For very low values of $\rho$ (extreme risk aversion), the CE is low, reflecting a high-risk premium. As $\rho$ increases, the DM becomes less risk-averse, and the CE value asymptotically approaches the risk-neutral value of $280.9 million. This curve quantifies the trade-off between risk and returns from the DM's subjective perspective.
- *Existence of a strategic tipping point:* The analysis identifies a specific risk tolerance threshold ($\rho\approx$$30 million) that represents a *strategic tipping point*. Below this threshold, the DM's aversion to uncertainty is so pronounced that they will never choose to simply continue the project; their optimal policy is always to make an active decision—most often to divest (75% of the time) and secure a guaranteed payoff. The "Continue" strategy, which maintains exposure to future volatility, is deemed too risky.

- *Convergence to risk-neutral policy:* Above the tipping point, the DM's preferences converge toward risk-neutrality. The optimal policy in the purple region {Buyout=25%, Continue=50%, Divest=25%} is identical to the policy derived from the standard risk-neutral valuation. This confirms the internal consistency of the model: as $\rho \to \infty$, the EUT-based valuation converges to the risk-neutral result, both in value and in optimal strategy.

- *Practical implication for decision-making:* This finding underscores that a singular, "correct" valuation or strategy does not exist. The value of the real options and the optimal course of action are entirely contingent on the risk appetite of the corporation or investor. Therefore, calibrating the risk tolerance parameter is not merely a theoretical exercise but a crucial step in ensuring that the strategic prescription aligns with the organization's fundamental risk posture

Fig. (**9**) presents the sensitivity of individual real option values to changes in the DM's risk tolerance ($\rho$). The chart plots the value of the Divest option, the Buyout option, and the Total Options Value (sum of both) across a spectrum of risk tolerance from zero to $1000 million. The following insights and trends can be inferred from this plot:

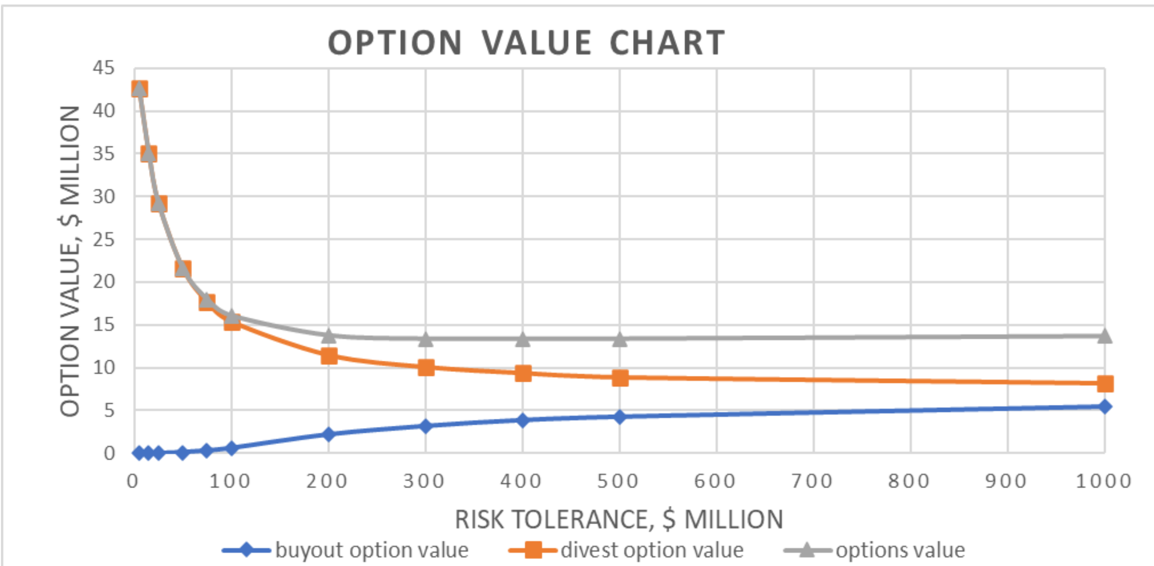


**Figure 9:** Sensitivity analysis of individual real option values to a risk-averse DM's risk preference for the modified BDH problem.

- *Divergent sensitivity of option values:* The most striking feature of the chart is the divergent behavior of the two option values as risk tolerance changes.
  - The value of the Divest option decreases monotonically as risk tolerance increases.
  - Conversely, the value of the *Buyout option increases* monotonically with higher risk tolerance. This divergence is fundamental and intuitive: the Divest option is an instrument of *downside protection* (a form of insurance), while the Buyout option is a tool for *upside capture* (a leveraged investment). Risk-averse DMs place a high premium on insurance, hence the high value of the divestment option at low risk tolerance. As aversion decreases, the value of this insurance declines. The opposite is true for the buyout option; its value as a speculative, growth-oriented investment is negligible to the highly risk-averse but becomes increasingly valuable as risk tolerance grows.

- *Additivity of option values***:** For this specific case, the *Total options value* (the sum of the two individual option values) remains relatively constant across the range of larger risk tolerance values. This suggests that the two options are largely *non-interacting*—the value of one does not materially affect the value of the other—and that their combined value is additive. This is a valuable, though project-specific, insight indicating that the options hedge different types of risk (downside vs. upside).

- *Convergence to risk-neutral values***:** As risk tolerance becomes very large (approaching infinity, i.e., risk-neutrality), the individual option values converge to their risk-neutral valuations. The chart shows the Buyout option value converging to a higher value and the Divest option to a lower one under these conditions, which is consistent with the standard real options theory that prioritizes upside potential in a risk-neutral world.

In conclusion, this sensitivity analysis reveals that the value of individual real options is not static but is a dynamic function of risk preference. Understanding this relationship is essential for making informed, rational investment decisions that are consistent with an organization's strategic risk appetite.

### 5.7. Scope and Limitations of the Case Study

The case study presented in this work represents a stylized producing-field investment under market-driven uncertainties. While this scenario effectively illustrates the mechanics and insights of the ROV–EUT framework, it is important to recognize that the numerical results are case-specific. Values such as the 75% divest likelihood under high risk aversion, the 50% continuation rate for a risk-neutral decision maker, or the risk-tolerance tipping point near 30 million USD arise directly from the project's cash-flow structure, volatility assumptions, and timing flexibility. These thresholds should therefore not be interpreted as universal. Different projects, particularly those with alternative payoff convexity, volatility characteristics, or abandonment options, would yield different numerical outcomes. The general qualitative pattern—that increasing risk aversion shifts optimal strategies toward more conservative or protective actions—remains robust across project types, but the exact quantitative boundaries depend on the underlying economic and uncertainty structure.

In addition, the current case study focuses on market uncertainties such as price and cost volatility that are central for developed producing assets. Petroleum investment decisions, however, often involve geological or exploration risks, including the probability of drilling success, reservoir uncertainty, and early-stage learning. These elements were intentionally omitted to maintain clarity and isolate the effect of attitude toward market risk. Nevertheless, integrating geological risk within the ROV–EUT framework represents a natural extension, as the utility-based approach can accommodate both discrete and continuous uncertainties. For example, in an exploration setting where probability of success is low and the distribution of outcomes is highly asymmetric, a risk-neutral evaluation may support aggressive drilling campaigns, whereas incorporating risk aversion through CEs could yield more selective exploration behavior. Such applications would further demonstrate the flexibility of the framework across the upstream value chain.

By explicitly acknowledging these limitations, the discussion clarifies where the findings can be extrapolated and highlights opportunities for applying the methodology to a broader set of project types. The intent of the case study is thus not to prescribe universally representative thresholds, but to demonstrate how risk preferences materially influence decisions when flexibility, uncertainty, and asymmetric payoffs interact.

### 5.8. Practical Complexity, Industry Adoption, and Implementation Guidance

Although the ROV–EUT framework offers a rigorous way to integrate managerial flexibility with corporate risk preferences, it is sometimes perceived as analytically demanding within industry settings. This perception largely reflects the combination of probabilistic modelling, option valuation, and utility-based evaluation. In practice, however, the majority of required inputs are already produced as part of standard petroleum decision processes. Cash-flow models, price scenarios, reservoir forecasts, and uncertainty ranges are routinely generated during corporate planning cycles, and these elements can be incorporated directly into the framework without requiring new data streams.

The extra modelling effort relates primarily to structuring the flexible decision problem and calculating certainty equivalents. These tasks can be performed using familiar tools, including spreadsheets, influence-diagram software, Monte Carlo simulators, MATLAB, or Python. Prior upstream applications of real-options analysis show clearly that such methods can be embedded within actual field development workflows using accessible computational environments. Examples include concept-selection studies under market and reservoir uncertainty [20, 36, 37], multi-factor evaluations of deep-water and offshore petroleum developments [5, 38], and practitioner-oriented real-options implementations using spreadsheet or basic analytic tools [39, 40]. These studies demonstrate that real-options modelling is compatible with the practical constraints, data availability, and decision processes of petroleum organizations.

Recent developments in the broader energy-economics literature also highlight increasing interest in explicitly modelling risk preferences. Studies incorporating risk aversion into investment timing, capacity expansion, power-sector planning, and hedging illustrate that risk attitudes can materially influence investment thresholds and optimal strategies [41-44]. Additional work on utility-based portfolio and commodity-price decision making further shows that risk-averse preferences systematically alter behaviour in energy markets [3, 45]. Collectively, this literature indicates that the integration of risk preferences within real-options structures is aligned with broader methodological trends and is particularly relevant for high-stakes petroleum investments.

These points together suggest that, although the ROV–EUT framework is analytically sophisticated, its practical adoption is entirely feasible within established corporate workflows. The approach is especially useful for large or strategically significant projects where flexibility and downside exposure have meaningful economic implications. Clarifying these considerations strengthens the case for industry uptake and shows that the framework can be implemented using tools and data that are already standard in upstream evaluation practices.

To further support real-world usability, it is helpful to express the ROV–EUT method as a workflow that mirrors how petroleum organizations typically conduct investment evaluations. The process begins with framing the flexible decision problem by identifying when expansion, deferral, acceleration, or abandonment options are available. Decision makers then identify the uncertainties that materially influence the project's economic performance, such as price volatility, operating-cost variability, and reservoir performance forecasts, all of which are standard outputs from existing planning processes. The organization's risk tolerance can be obtained from historical investment behaviour, corporate finance guidelines, or standard utility-elicitation procedures. These components feed into a probabilistic cash-flow model, typically using Monte Carlo simulation or scenario-based modelling. Once uncertainties are linked to the real-options structure, the decision tree is evaluated using certainty equivalents, allowing strategies to be ranked in a way that is consistent with corporate risk preferences. Because these steps parallel existing industry workflows, the methodology can be adopted without requiring new infrastructure or specialized software.

It is also important to clarify how the proposed ROV–EUT methodology differs conceptually from conventional risk-neutral real-options analysis. Standard ROV maximizes the expected monetary value (EV) of future payoffs, assuming indifference to risk. When expected utility theory is incorporated, the evaluation metric shifts from EV to the certainty equivalent (CE), which represents the risk-adjusted monetary value of a risky payoff. All decisions are therefore based on maximizing CE rather than EV. This distinction is crucial because the CE accounts for downside exposure and embeds the firm's risk tolerance; as a result, a strategy that is optimal under risk neutrality may become suboptimal once risk aversion is considered. Including this clarification helps illustrate why optimal decisions differ across risk profiles and how corporate preferences materially shape investment behaviour.

# 6. Conclusion and Future Work

## 6.1. Conclusion

This study developed and demonstrated an integrated decision-making framework that combines ROV with EUT to evaluate investments under uncertainty, with explicit consideration of managerial risk preferences. The methodology was applied to a representative oil production project case study, characterized by market-driven uncertainties in oil price and operating costs, and enhanced with two critical real options: the option to divest and the option to buy out additional partnership.

The results affirm that this combined approach offers petroleum engineers and investment analysts a more comprehensive and practically relevant decision-support tool than conventional methods. By quantifying how risk aversion influences both project valuation and strategic choice, the framework bridges the gap between theoretical market-based valuation and real-world corporate decision-making. It provides not only a value assessment but also a clear, risk-aware policy recommendation tailored to the specific risk posture of the decision-making entity.

The main conclusions derived from this analysis are:

- *Real options create significant value*: The inclusion of flexibility, in the form of the option to divest or buy out, substantially increases the project's value compared to a passive management strategy.
- *Risk attitude dictates strategy*: The optimal investment strategy is highly sensitive to the DM's risk tolerance. A risk-neutral party frequently chooses to continue operations, while a risk-averse party prefers the certainty of divestment.
- *Option values are sensitive to risk preference*: The value of individual real options changes dramatically with risk aversion. The value of the divest option (downside protection) increases for more risk-averse entities (lower risk tolerance values), while the value of the buyout option (upside potential) decreases.
- *A strategic tipping point exists*: The analysis identified a specific risk tolerance threshold that triggers a shift in the optimal policy, demonstrating that there is no universal "correct" decision.
- *EUT provides a robust framework for risk analysis*: EUT offers a consistent and transparent method for incorporating subjective risk preferences into quantitative investment models, leading to decisions that are rational and aligned with corporate risk appetite.
- *There is a strategic portfolio of options***:** This analysis reframes the real options not as a single flexibility value but as a *portfolio of distinct strategic instruments*. A company's risk profile directly determines which "instrument" in its portfolio is most valuable. A conservative firm will rightly value its abandonment option most highly, while an aggressive firm will derive more value from its growth option.
- *Objective valuation is implemented and compared against a subjective value***:** The risk-neutral value represents an objective, market-consistent price for the options. However, the conducted study clearly demonstrates that their *subjective value* to a particular DM can differ dramatically. This explains why two companies might disagree on the fair price for buying or selling such options, based solely on their differing risk appetites.

### 6.2. Directions for Future Research

Building upon the methodology and findings of this study, several promising directions for future research emerge:

- Multi-option and interacting options analysis: Future work could explore more complex option structures, including sequential, mutually exclusive, or compound real options. Investigating how interactions between multiple options—such as the synergy between abandonment and expansion options—affect value and optimal strategy under different risk preferences would yield deeper managerial insights.
- Alternative utility functions and risk preferences: While the exponential utility function offers computational advantages, future studies could examine other forms (e.g., hyperbolic absolute risk aversion) or state-dependent risk preferences. Modeling organizational risk tolerance as a dynamic rather than static parameter could also better reflect real-world decision-making.
- Behavioral and empirical validation: Conducting empirical studies or behavioral experiments with industry professionals to calibrate risk tolerance parameters and validate the model's prescriptions could strengthen the practical relevance of the approach.
- Machine learning and advanced numerical methods: Integrating machine learning techniques for volatility forecasting or using more efficient numerical methods (e.g., LSMC for multi-factor options) could improve the scalability and accuracy of the analysis for high-dimensional problems.

These avenues would not only extend the theoretical foundations of risk-informed real options analysis but also enhance its applicability and adoption in energy investment practice.

## Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

## References

[1] Smith JE, McCardle KF. Valuing oil properties: Integrating option pricing and decision analysis approaches. Oper Res. 1998; 46(2): 198-217. https://doi.org/10.1287/opre.46.2.198

[2] Trigeorgis L. Real Options: Managerial Flexibility and Strategy in Resource Allocation. Cambridge: MIT Press; 1996.

[3] Dixit AK, Pindyck RS. Investment under Uncertainty. Princeton: Princeton University Press; 1994. https://doi.org/10.1515/9781400830176

[4] Copeland T, Antikarov V. Real Options: A Practitioner's Guide. New York: Texere; 2001.

[5] Brandão LE, Dyer JS, Hahn WJ. Using binomial decision trees to solve real option valuation problems. Decis Anal. 2005; 2(2): 69-88. https://doi.org/10.1287/deca.1050.0040

[6] Dias MAG. Real options theory for petroleum: A review. Energy Econ. 2019; 81: 305-23.

[7] Ahmadi R, Bratvold RB. An exposition of least square Monte Carlo approach for real options valuation. J Petrol Sci Eng. 2023; 232: 111230. https://doi.org/10.1016/j.petrol.2022.111230

[8] Bratvold RB, Begg SH. Making Good Decisions. Society of Petroleum Engineers; 2010. https://doi.org/10.2118/9781555632588

[9] Hull JC. Risk Management and Financial Institutions. 4th ed. Wiley; 2015.

[10] von Neumann J, Morgenstern O. Theory of Games and Economic Behavior. 2nd ed. Princeton: Princeton University Press; 1947.

[11] Pratt JW. Risk aversion in the small and in the large. Econometrica. 1964; 32(1-2): 122-36.

[12] Clemen RT, Reilly T. Making Hard Decisions with Decision Tools. 3rd ed. Cengage Learning; 2014.

[13] Costa Lima GA, Suslick SB. Investment decision in oil and gas projects using real option and risk tolerance models. Int J Oil Gas Coal Technol. 2008; 1(1/2): 3-23. https://doi.org/10.1504/IJOGCT.2008.016729

[14] Aspinall D, Bowe M, Kretzschmar GL. The practical use of real options in commodity projects: Evidence from mining and energy. Resour Policy. 2023; 82: 103608. https://doi.org/10.1016/j.resourpol.2023.103608

[15] Kennedy J, Davies S. Barriers to the adoption of probabilistic methods in petroleum decision making: An empirical study. J Petrol Sci Eng. 2021; 205: 108870. https://doi.org/10.1016/j.petrol.2021.108870

[16] MacMillan I. The adoption gap in real options valuation: Lessons from the oil and gas industry. Energy Econ. 2009; 31(1): 43-52. https://doi.org/10.1016/j.eneco.2008.08.002

[17] Lee M. Why probabilistic thinking fails in engineering practice: A case study of oil and gas projects. SPE Econ Manag. 2015; 7(4): 156-64. https://doi.org/10.2118/167146-PA

[18] Sarumpaet S, Purwanto A, Wibowo S. Real options valuation of oilfield projects under price and technical uncertainty: A binomial lattice approach. J Petrol Sci Eng. 2025; 237: 112094. https://doi.org/10.1016/j.petrol.2025.112094

[19] Cox JC, Ross SA, Rubinstein M. Option pricing: A simplified approach. J Financ Econ. 1979; 7(3): 229-63. https://doi.org/10.1016/0304-405X(79)90015-1

[20] Smith JE. Alternative approaches for solving real-options problems. Decis Anal. 2005; 2(2): 89-102. https://doi.org/10.1287/deca.1050.0041

[21] Fedorov A, Shadrina E, Chudinova E. Application of real options valuation for marginal oilfield development under uncertainty. Resour Policy. 2020; 66: 101629. https://doi.org/10.1016/j.resourpol.2020.101629

[22] Martins F, Antunes CH, Brito M. Valuation of investment in wave energy projects using real options: The case of Portugal. Energy. 2021; 219: 119581. https://doi.org/10.1016/j.energy.2020.119581

[23] Schachter J, Uryasev S, Zabarankin M. Flexible manufacturing and real options: Input–output switching under uncertainty. Eur J Oper Res. 2019; 277(1): 331-43.

[24] Hull JC. Options, Futures, and Other Derivatives. 10th ed. Pearson; 2018.

[25] Longstaff FA, Schwartz ES. Valuing American options by simulation: A simple least-squares approach. Rev Financ Stud. 2001; 14(1): 113-47. https://doi.org/10.1093/rfs/14.1.113

[26] Black F, Scholes M. The pricing of options and corporate liabilities. J Polit Econ. 1973; 81(3): 637-54. https://doi.org/10.1086/260062

[27] Wood DA, Khosravanian R. Exponential utility functions aid upstream decision making. J Nat Gas Sci Eng. 2015; 27: 1482-94. https://doi.org/10.1016/j.jngse.2015.10.012

[28] Velasquez HG, Deutsch CV. Decision making in the presence of geological uncertainty. Geostatistics Lessons. 2021.

[29] Markowitz H. Portfolio selection. J Finance. 1952; 7(1): 77-91. https://doi.org/10.1111/j.1540-6261.1952.tb01525.x

[30] Artzner P, Delbaen F, Eber JM, Heath D. Coherent measures of risk. Math Finance. 1999; 9(3): 203-28. https://doi.org/10.1111/1467-9965.00068

[31] Bernoulli D. Specimen theoriae novae de mensura sortis. Commentarii Academiae Scientiarum Imperialis Petropolitanae. 1738; 5: 175-92.

[32] Stearns C, Bernoulli D. Bernoulli's theory of utility. J Risk Uncertain. 2000; 21: 103-20.

[33] Edwards W. Utility theories: Measurement and applications. In: Edwards W, Ed. Handbook of Decision Analysis. 1992. https://doi.org/10.1007/978-94-011-2952-7

[34] Howard RA, Abbas AE. Foundations of Decision Analysis. 2nd ed. Pearson; 2016.

[35] Ahmadi R, Bratvold RB. Real options valuation under risk-neutrality and risk aversion for an oil production project. In: Energy Norway Conference; 2022.

[36] Dias MAG. Real options analysis in petroleum projects: Investment opportunities at different stages of field development. Energy Policy. 2004; 32(2): 209-15. https://doi.org/10.1016/S0301-4215(02)00297-0

[37] Jafarizadeh B, Alizadeh N, Reynolds AC. Real options and value of information in petroleum reservoir management. SPE Econ Manag. 2010; 2(4): 157-67. https://doi.org/10.2118/124905-PA

[38] Söderholm P, Wårell L. Market uncertainties and investment in petroleum exploration. Energy Econ. 2011; 33(3): 471-8. https://doi.org/10.1016/j.eneco.2010.12.006

[39] Mun J. Real Options Analysis: Tools and Techniques for Valuing Strategic Investments and Decisions. 2nd ed. Hoboken: Wiley; 2006.

[40] Kodukula P, Papudesu C. Project Valuation Using Real Options: A Practitioner's Guide. Fort Lauderdale: J. Ross Publishing; 2006.

[41] Siddiqui AS, Fleten SE. How to treat risk in stochastic capacity expansion planning: The case of renewable energy. Energy Econ. 2010; 32(2): 415-25. https://doi.org/10.1016/j.eneco.2009.07.004

[42] Kazempour J, Conejo AJ. Electricity market clearing under uncertainty and risk aversion. IEEE Trans Power Syst. 2015; 30(5): 2407-16. https://doi.org/10.1109/TPWRS.2014.2360104

[43] Jin S, Ryan SM, Watson JP. What am I worth? The value of risk preferences in stochastic unit commitment. Oper Res. 2011; 59(3): 733-45. https://doi.org/10.1287/opre.1100.0890

[44] Miao J, Wang N. Investment, consumption, and hedging under incomplete markets. J Financ Econ. 2007; 86(3): 608-42. https://doi.org/10.1016/j.jfineco.2006.10.006

[45] Ekeland I, Karp L, Sumaila UR. Equilibrium resource management with risk-averse agents. J Environ Econ Manage. 2015; 69: 1-17. https://doi.org/10.1016/j.jeem.2014.10.003

[46] Keeney RL, Raiffa H. Decisions with Multiple Objectives: Preferences and Value Tradeoffs. Cambridge: Cambridge University Press; 1993. https://doi.org/10.1017/CBO9781139174084

[47] Farquhar PH. Utility assessment methods. Manage Sci. 1984; 30(11): 1283-300. https://doi.org/10.1287/mnsc.30.11.1283

# Appendix A

## A.1. Practical Calibration of the Risk-Tolerance Parameter

Assigning an appropriate value for the risk-tolerance parameter ($\rho$) is a critical step for applying the ROV–EUT framework in practice. In real organizations, $\rho$ can be estimated through several complementary approaches. One option is to analyze historical investment decisions and infer the level of downside exposure that management has historically accepted or rejected, thereby revealing an implicit risk-tolerance range. Alternatively, many companies maintain internal guidelines on hurdle rates, portfolio exposure limits, or maximum acceptable downside losses; these can be used to calibrate $\rho$ so that the implied certainty-equivalent values align with established corporate behaviour. In situations where direct organizational data are unavailable, standard utility-elicitation techniques—such as certainty-equivalent elicitation [46] or probability-equivalent assessments [47]—can be applied through structured interviews with senior decision makers to derive an explicit risk-tolerance estimate. These methods are widely used in decision analysis and provide practical pathways for grounding $\rho$ in observable preferences. Providing this guidance helps bridge the gap between theoretical formulation and real-world implementation.

## A.2. EUT Background Example

We consider a petroleum exploration project in which a DM faces uncertainty regarding the outcome of a drilling venture. The project has two possible results: a *success*, yielding a substantial payoff if oil is discovered, and a *failure*, yielding no financial return if the well is dry. The probability of success is relatively low, reflecting the geological and operational risks inherent in exploration activities.

The DM must evaluate the project while accounting for their individual risk preferences—risk-averse, risk-neutral, or risk-seeking—since the EMV alone does not capture how uncertainty affects project attractiveness. The goal is to determine the CE for the project under different risk attitudes, providing a consistent, interpretable measure of the project's value for decision-making. The possible outcomes along with their corresponding probabilities for the above project are listed in Table **A1**.

**Table A1: Possible outcomes and corresponding probabilities of the oil exploration project**

| Outcome | Probability | Monetary Value (M$) |
|---|---|---|
| Success | 0.2 | 100 |
| Failure | 0.8 | 0 |

The EV of this project is:

$$EV = 0.2 \times 100 + 0.8 \times 0 = 20\ M\$ \tag{A1}$$

We evaluate this prospect for three types of DMs, using the exponential utility formulation of Eq. 6 using $\rho$ values of 30 and -30 M$ for risk-aversion and risk-seeking attitudes, respectively.

***Step 1: compute EU***

- Risk-neutral:

$$EU = 0.2 \cdot 100 + 0.8 \cdot 0 = 20\ M\$ \tag{A2}$$

- Risk-averse (ρ=30):

$$EU = 0.2 \cdot 30 \left(1 - e^{\left\{-\frac{100}{30}\right\}}\right) + 0.8 \cdot 30 \left(1 - e^{\left\{-\frac{0}{30}\right\}}\right) \approx 5.78\ M\$ \tag{A3}$$

- Risk-seeking (ρ=−30)

$$EU = 0.2 \cdot (-30)\left(1 - e^{\left\{\frac{100}{30}\right\}}\right) + 0.8 \cdot (-30)\left(1 - e^{\left\{\frac{0}{30}\right\}}\right) \approx 162\ M\$ \quad \text{(A4)}$$

***Step 2: calculate CE***

The CE is obtained by inverting the utility function:

$$CE = -\rho\, \mathbf{ln}(1 - \rho . EU) \quad \text{(A5)}$$

- Risk-neutral:

$$CE = EV = 20\ M\$ \quad \text{(A6)}$$

- Risk-averse:

$$CE = -30\, \mathbf{ln}(1 - (5.78 \times 30)) \approx 6.6\ M\$ \quad \text{(A7)}$$

- Risk-seeking:

$$CE = -(-30)\, \mathbf{ln}(1 - (162 \times (-30))) \approx 53.5\ M\$ \quad \text{(A8)}$$

***Step 3: interpretation***

- The risk-neutral DM evaluates the project according to EV; the CE equals 20 M$ and lies within the range of possible payoffs.
- The risk-averse DM assigns a CE of 6.6 M$, reflecting their preference to avoid the high probability of failure.
- The risk-seeking DM assigns a CE of 53.5 M$, above the EV but below the maximum possible payoff (100 M$), reflecting a preference for high upside potential.

This example demonstrates how CE provides a consistent, monetary measure of project attractiveness across different risk attitudes, allowing rational decision-making under uncertainty. Importantly, by using the exponential utility formulation with bounded *ρ*, the CE remains realistic and interpretable, even for risk-seeking DMs.

### A.3. Detailed Monte Carlo Simulation Procedure

The combined effect of all uncertainties on the project's value is captured by a single parameter: the project volatility (*σ*). This is estimated by running a MCS on the project's free cash flows under the passive scenario. The passive scenario assumes that no active decisions or management interventions are made during the project's lifetime. The MCS involves the following steps:

1. *Generate Random Samples*: For each uncertainty modeled as a stochastic process, generate a large number of random samples based on the chosen distribution (e.g., GBM).
2. *Calculate Free Cash Flows*: Use the random samples to calculate the project's free cash flows for each period in the project's lifetime.
3. *Calculate Project Value*: Discount the free cash flows back to the present to obtain the project's value for each simulation run.
4. *Repeat*: Repeat steps 1-3 a large number of times (e.g., 10,000 or more) to generate a distribution of project values.

The resulting distribution of project values represents the range of possible outcomes for the project under the passive scenario, given the uncertainties modeled.

The standard deviation of the logarithmic returns of the resulting distribution of project values yields the annualized project volatility, which is essential for building the binomial tree. The logarithmic return (*z*) between two consecutive project values is calculated as:

$$z = \ln\left(\frac{V_1}{\overline{V_0}}\right) \tag{A9}$$

where $\overline{V_0}$ is the project value at time 0, and $V_1$ is the project value at time 1. The standard deviation of the logarithmic returns is then calculated. This represents the volatility of the project's returns over the time period used for calculating the returns. The volatility is annualized by multiplying the standard deviation by the square root of the number of time periods in a year. If the time period used for calculating the returns is one month, then the annualized volatility is calculated as:

$$\sigma = std(z).\sqrt{\Delta t} \tag{A10}$$

where *std(z)* is the standard deviation of the logarithmic returns, and Δt represents the length of the time period (e.g., 1/12 for monthly data). The resulting annualized project volatility (σ) represents the overall volatility of the project's value, taking into account the combined effect of all the uncertainties modeled.