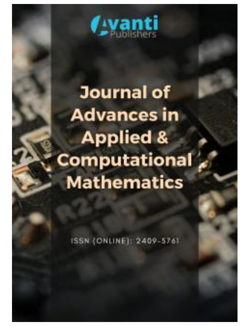
Published by Avanti Publishers

Journal of Advances in Applied & Computational Mathematics

ISSN (online): 2409-5761



Lateral Free Vibration of Micro-Rods Using a Nonlocal Continuum Approach

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ARTICLE INFO

Article Type: Research Article

Keywords:
Micro-rod
Free vibration
Vibration mode
Natural frequency
Nonlocal continuum

Timeline:
Received: September 13, 2022
Accepted: October 18, 2022
Published: October 24, 2022

Citation: Xie F, Zhang N, Chen C, Li C. Lateral free vibration of micro-rods using a nonlocal continuum approach. J Adv App Comput Math. 2022; 9: 157-67.

DOI: https://doi.org/10.15377/2409-5761.2022.09.12

ABSTRACT

The lateral free vibration of micro-rods initially subjected to axial loads based on a nonlocal continuum theory is considered. The effects of nonlocal long-range interaction fields on the natural frequencies and vibration modes are examined. A simply supported micro-rod is taken as an example; the linear vibration responses are observed by two different methods, including the separation of variables and multiple scales analysis. The relations between the vibration mode and dimensionless coordinate and the relations between natural frequencies and nonlocal parameters are analyzed and discussed in detail. The numerical comparison shows that the theoretical results by two different approaches have a good agreement, which validates the present micro-rod model that can be used as a component of the micro-electromechanical system.

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1. Introduction

Recently, dynamic behaviors, especially the vibration and control of micro-structures, have become a hot topic because of the application prospects of micro-electromechanical systems (MEMS) and other related micro-robot components. Motivated by this, we consider the linear vibration of a simply supported micro-rod using a non-classical continuum theory. In the previous literature, either only the vibration of the classical macro-rod was considered, or the dynamic characteristics of the micro-rod were studied based on the classical continuum theory.

For the small-size solids mechanics, the surface effect and size effect play essential roles in mechanical characteristics. He *et al*. [1, 2] presented much research on the surface effect in the past years. On the other hand, the nonlocal effect is one of the most critical common side effects, which assumes that the stress at a point is a function of strains at all points in the domain. Such the theory contains information about the forces between atoms or molecules, and the internal length scale is introduced into the constitutive relations as an inherent material parameter. On the contrary, most classical mechanics theories based on elastic constitutive relation regard the stress at a point as a function of strain at only that point. The nonlocal continuum theory and its embryonic form were initiated by Eringen, Edelen, and Kroner [3-5] in the 1960s-1970s. The so-called nonlocal continuum refers to a particular non-classical continuum. As we know, classical mechanical theories are based on the continuity hypothesis. However, micro/nanoscale materials and structures are usually considered discontinuous. The atomic simulation plays an important role in this respect, and some meaningful results of discontinuous objects can be obtained via the atomic simulation at a small scale. After that, the concept of a non-classical continuum is proposed by combining the classical continuum with atomic simulation. Among the non-classical continuum theories, the most common is the nonlocal continuum, which considers the influence of the long-range interaction between atoms or molecules. In a certain neighborhood, the stress and strain relationship at one point is related to the stress and strain of all the other points, which of course, does not belong to discontinuity but also does not belong to the classical continuum. This is called the nonlocal continuum. In other words, the global performance of the research object is different from the local performance, so the local can no longer represent the whole.

Currently, the nonlocal continuum theory has been applied in micro-mechanics more and more widely [6-15]. For example, recently, Awrejcewicz *et al*. [15] investigated the vibration of rectangular micro/nanoplates using the nonlocal theory. The geometric non-linearity was considered, and a multiple-scale method was applied to demonstrate the small-scale effects of the micro/nanoplates. An impressive example occurs in fracture mechanics. When the classical continuum theory is used to analyze the stress field at the crack tip, the result shows that it is singular but unreasonable and difficult to explain physically. While using the nonlocal continuum theory, the result shows that the stress field at the crack tip is non-singular, or no stress singularity is observed at the crack tip, which is a reasonable result.

This paper establishes the problem model and then describes using a non-dimensional fourth-order partial differential equation based on the nonlocal continuum theory. Two different methods, namely separation of variables and multiple scales analysis, are applied to the governing equation. Of course, many other numerical methods [16-23] can deal with nonlinear partial differential equations and other more complex models. For the linear vibration model of micro-rods in this study, the above two methods are enough to solve and the results with sufficient accuracy can be obtained. It is found that the initial tension and nonlocal parameter play significant roles in the vibration behavior of such a simply supported micro-rod. Results by two methods are compared and discussed in detail.

2. Mechanics Modeling

Consider a simply supported micro-rod with initial axial tension P at both ends. The dynamical equilibrium for an element of the micro-rod is illustrated in Figure **1**. It is noticed that only the Euler-Bernoulli beam model is adopted here. This is because the present micro-rod model is a slender rod. The Euler-Bernoulli beam model is sufficient for the current problem. Of course, the Timoshenko beam model is more accurate than the Euler-Bernoulli beam model because it takes into account the moment of inertia and shear effects of the structure. The Timoshenko beam model and other high-order shear beam models are generally more effective in predicting the mechanical behaviors of short or thick beams. For a slender micro-rod considered in the present study, the Euler-Bernoulli beam model works and other beam models are not required.

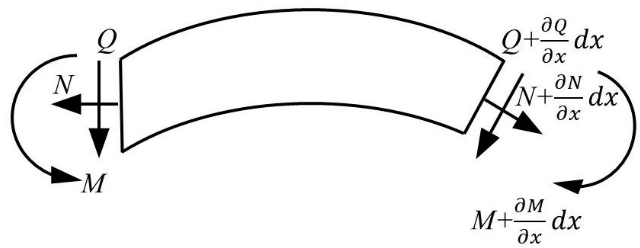

Figure 1: Mechanical analysis for a micro-rod element.

The figure above M denotes the bending moment, N internal axial force, Q shear force, x axial coordinate, and z transverse coordinate. Only the small deformation is considered for linear vibration of a supported micro-rod. The equilibrium equation of the element concerning the transverse direction can be obtained based on the D' Alembert principle as

$$\frac{\partial^2 M}{\partial x^2} - P\frac{\partial^2 w}{\partial x^2} + \rho\frac{\partial^2 w}{\partial t^2} = 0 \tag{1}$$

where w is the lateral deformation.

According to the nonlocal continuum theory, the following relation between bending moment and lateral deformation is governed by [24]

$$M - (e_0 a)^2\frac{\partial^2 M}{\partial x^2} = -EI\frac{\partial^2 w}{\partial x^2} \tag{2}$$

where EI is the flexural stiffness. From Eqs. (1) and (2), the following partial differential equation that governs the vibration motion for a simply supported micro-rod subjected to an initial axial tension P can be derived as

$$P\frac{\partial^2 w}{\partial x^2} - \rho\frac{\partial^2 w}{\partial t^2} - (e_0 a)^2\left(P\frac{\partial^4 w}{\partial x^4} - \rho\frac{\partial^4 w}{\partial x^2 \partial t^2}\right) = -EI\frac{\partial^4 w}{\partial x^4} \tag{3}$$

The parameters $e_0 a$ associated with the nonlocal continuum theory can be contained using a known theoretical method [25].

Introduce the following non-dimensional parameters and variables

$$\bar{x} = \frac{x}{L}, \bar{w} = \frac{w}{L}, \bar{t} = t\sqrt{\frac{EI}{\rho L^4}}, \tau = \frac{e_0 a}{L}, \bar{P} = \frac{PL^2}{EI} \tag{4}$$

where τ is a nanoscale nonlocal parameter that denotes the nonlocal effects and $\bar{P}$ the dimensionless initial tension. Then Eq. (3) becomes

$$\bar{P}\frac{\partial^2 \bar{w}}{\partial \bar{x}^2} - \frac{\partial^2 \bar{w}}{\partial \bar{t}^2} + \tau^2\frac{\partial^4 \bar{w}}{\partial \bar{x}^2 \partial \bar{t}^2} - (\bar{P}\tau^2 - 1)\frac{\partial^4 \bar{w}}{\partial \bar{x}^4} = 0 \tag{5}$$

3. The Separation of Variables

For linear free vibration of a simply supported micro-rod, the method of separation of variables can be applied directly. So the solution of Eq. (5) can be assumed as

$$\bar{w}(\bar{x},\bar{t}) = \sum_{i=1}^{n} \phi_i(\bar{x}) q_i(\bar{t}) \tag{6}$$

where ϕ_i is the dimensionless vibration mode, while q_i is the temporal function. Substitution of Eq. (6) into the equation of motion (5) and separation of the two variables in the resulting equation, one obtains the two following ordinary differential equations as

$$\frac{d^2 q}{d\bar{t}^2} + \omega_n^2 q = 0 \tag{7a}$$

$$(\bar{P}\tau^2 - 1)\frac{d^4\phi}{d\bar{x}^4} + (\omega_n^2\tau^2 - \bar{P})\frac{d^2\phi}{d\bar{x}^2} - \omega_n^2\phi = 0 \tag{7b}$$

where ω_n is the dimensionless natural frequency. Note that only the linear free vibration is considered in this paper. Hence it is the free vibration natural frequency herein. When a structure vibrates freely, its displacement changes with time according to the sine or cosine function. The vibration frequency is independent of initial conditions but only related to the intrinsic characteristics of the structure (such as mass, shape, material, etc.), which is called the natural frequency.

The ordinary differential equation (7b) belongs to the fourth-order differential equation in form. Let $\phi = e^{r\bar{x}}$ and substitute it into Eq. (7b); one can obtain

$$(\bar{P}\tau^2 - 1)r^4 e^{r\bar{x}} + (\omega_n^2\tau^2 - \bar{P})r^2 e^{r\bar{x}} - \omega_n^2 e^{r\bar{x}} = 0 \tag{8}$$

According to the ordinary differential equation theory, the characteristic equation of Eq. (7b) is

$$(\bar{P}\tau^2 - 1)r^4 + (\omega_n^2\tau^2 - \bar{P})r^2 - \omega_n^2 = 0 \tag{9}$$

Assuming $r^2 = m$, the characteristic equation turns into

$$(\bar{P}\tau^2 - 1)m^2 + (\omega_n^2\tau^2 - \bar{P})m - \omega_n^2 = 0 \tag{10}$$

The solution of Eq. (10) can be determined as

$$m_1 = \frac{\bar{P} - \omega_n^2\tau^2 + \sqrt{(\bar{P} + \omega_n^2\tau^2)^2 - 4\omega_n^2}}{2(\bar{P}\tau^2 - 1)} \tag{11a}$$

$$m_2 = \frac{\bar{P} - \omega_n^2\tau^2 - \sqrt{(\bar{P} + \omega_n^2\tau^2)^2 - 4\omega_n^2}}{2(\bar{P}\tau^2 - 1)} \tag{11b}$$

where $(\bar{P} + \omega_n^2\tau^2)^2 - 4\omega_n^2 > 0$ holds because the vibration of micro-structures generally belongs to high-frequency vibration. For example, the natural vibration frequency of carbon nanotubes can reach the order of THz. It is not difficult to prove that $m_1 > 0$. For m_2, whether $\bar{P}\tau^2 > 1$ or $\bar{P}\tau^2 < 1$, $m_2 < 0$ is always valid, but it requires $\bar{P}\tau^2 \neq 1$. Therefore, it implies that the initial axial tension cannot be arbitrary, and a certain relationship must be satisfied between the tension and the nonlocal parameter. That is, the intrinsic physical parameters put forward requirements for the external parameters of the micro-rod.

Consequently, the characteristic equation (9) has two different real roots and two imaginary conjugate roots. Accordingly, the general solution of the fourth-order ordinary differential equation (7b) can be expressed as

$$\phi(\bar{x}) = C_1 \sin(\alpha\bar{x}) + C_2 \cos(\alpha\bar{x}) + C_3 \sinh(\beta\bar{x}) + C_4 \cosh(\beta\bar{x}) \tag{12}$$

where

$$\alpha = \sqrt{\frac{-\bar{P} + \omega_n^2\tau^2 + \sqrt{(\bar{P} + \omega_n^2\tau^2)^2 - 4\omega_n^2}}{2(\bar{P}\tau^2 - 1)}} \tag{13a}$$

$$\beta = \sqrt{\frac{\bar{P} - \omega_n^2\tau^2 + \sqrt{(\bar{P} + \omega_n^2\tau^2)^2 - 4\omega_n^2}}{2(\bar{P}\tau^2 - 1)}} \tag{13b}$$

The boundary conditions of the simply supported micro-rod require

$$\phi(0) = 0 \quad ; \quad \phi(1) = 0 \quad ; \quad \frac{d^2\phi}{\partial \bar{x}^2}(0) = 0 \quad ; \quad \frac{d^2\phi}{\partial \bar{x}^2}(1) = 0 \tag{14}$$

Substituting Eq. (12) into (14) and making the coefficient matrix determinant of the resulting algebra equation vanishing to have non-trivial solutions, we can obtain the n -order natural frequency as

$$\omega_n = \sqrt{\frac{(\bar{P}\tau^2 - 1)n^4\pi^4 + \bar{P}n^2\pi^2}{1 + n^2\pi^2\tau^2}} \tag{15}$$

and the vibration mode function as

$$\phi(\bar{x}) = C_1\, sin(n\pi\bar{x}) \tag{16}$$

The relationship between the first three order vibration modes and dimensionless coordinate $\bar{x}$, the first three order natural frequencies, and nanoscale nonlocal parameter τ are shown in Figures **2** and **3**, respectively. It can be seen from Figure **2** that the vibration mode changes with the dimensionless coordinate in the way of a similar sine law but not a complete sine function. This is because the vibration mode of the micro-rod is different from that of a macro-rod based on the classical vibration mechanics theory. Under the influence of the nonlocal parameter, the vibration mode changes compared with the simple harmonic vibration of the classical rod. It is also observed that both the vibration modes and natural frequencies are influenced by the nonlocal parameter, where the dimensionless initial tension is assumed, as $\bar{P} = 35$ in Figure **3**. Besides, the natural frequency increases with the increase of nonlocal parameters. However, the influence of the nonlocal parameter on the first-order natural frequency (fundamental frequencies) is less evident than that of the second and third-order, which indicates that

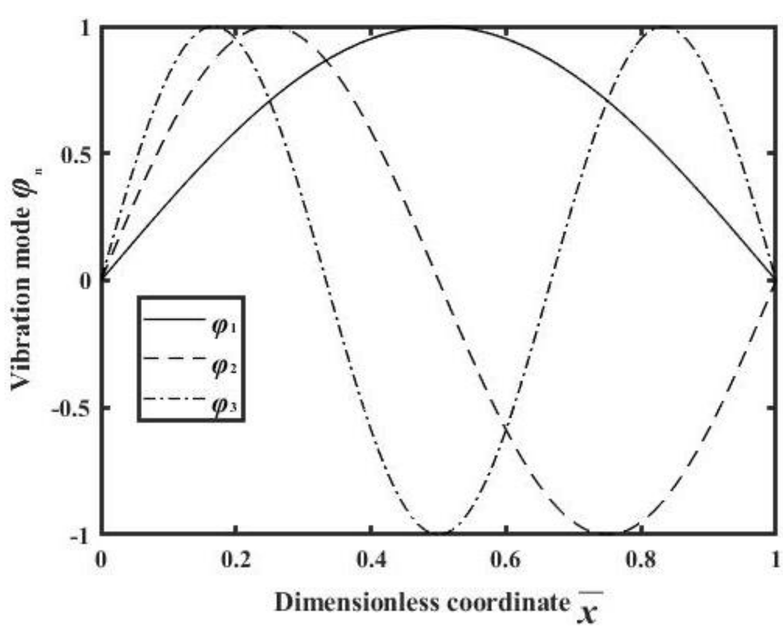

Figure 2: Relations between vibration mode and dimensionless coordinate.

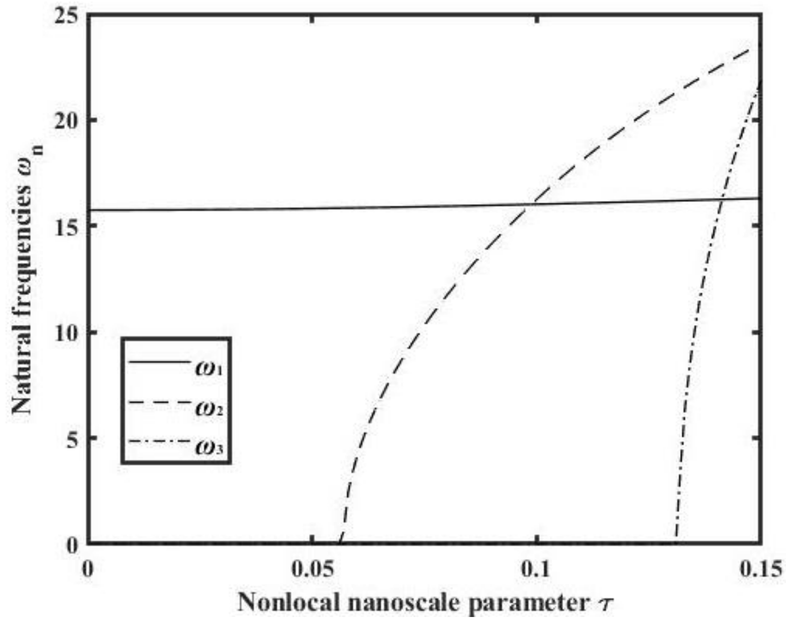

Figure 3: Effects of nonlocal parameters on natural frequencies.

the effect of the nonlocal parameter should be considered more in the higher-order vibration modes of micro-structures. Effects of the dimensionless initial tension on the fundamental frequencies are illustrated in Figure **4**. Natural frequencies increase with stronger nonlocal effects or larger initial tension in Figure **4**.

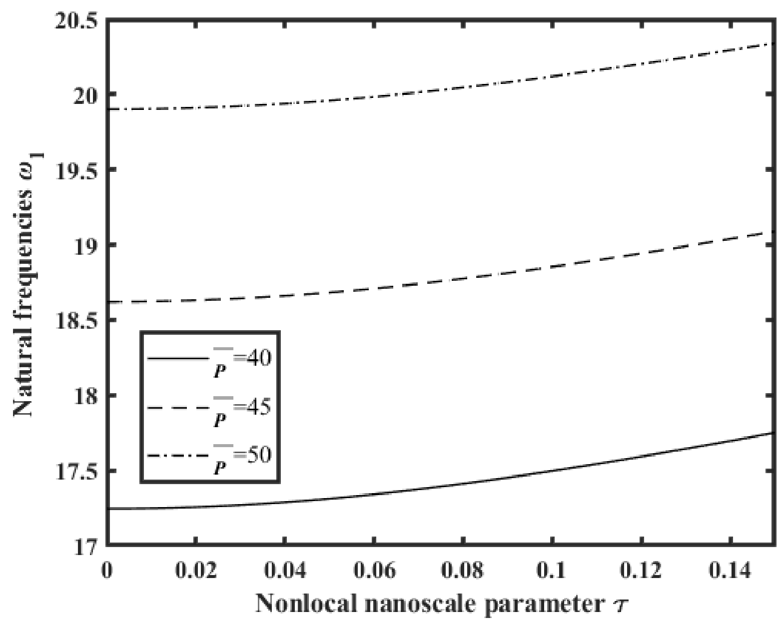

Figure 4: Effects of dimensionless initial tension on natural frequencies.

From the above Eqs. (6), (7a), (15), and (16) can be deduced as the result.

$$\bar{w}(\bar{x},\bar{t}) = \sum_{n=1}^{\infty} sin(n\pi\bar{x})\, e^{i\sqrt{\frac{(\bar{P}\tau^2-1)n^4\pi^4+\bar{P}n^2\pi^2}{1+n^2\pi^2\tau^2}}\bar{t}} \tag{17}$$

where we suppose $C_1 = 1$. Figure **5** shows the evolvement of the first-term approximation of lateral displacement and non-dimensional axial coordinate and time coordinate. It is demonstrated that the lateral displacement varies remarkably with changing the dimensionless time and dimensionless coordinate. The changing track is close to the trigonometric function but not entirely in the form of sine or cosine curves due to the inherent nonlocal parameter in a micro-rod.

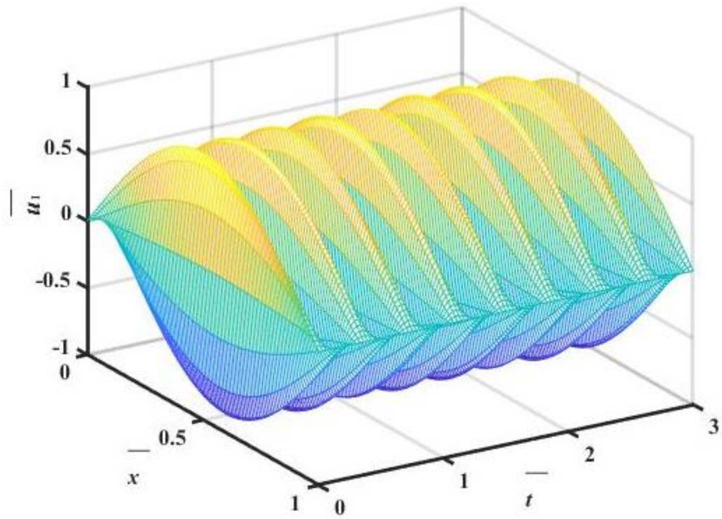

Figure 5: The evolvement of the lateral displacement.

4. The Method of Multiple Scales

In this section, the method of multiple scales will be applied to Eq. (5) directly to determine the natural frequencies, which have been shown that affected by nonlocal nanoscale nonlocal parameters and initial tension. Suppose $\varepsilon = \tau^2$ it is a small dimensionless variable and then Eq. (5) becomes

$$\bar{P}\frac{\partial^2 \bar{w}}{\partial \bar{x}^2} - \frac{\partial^2 \bar{w}}{\partial \bar{t}^2} + \varepsilon \frac{\partial^4 \bar{w}}{\partial \bar{x}^2 \partial \bar{t}^2} - (\bar{P}\varepsilon - 1)\frac{\partial^4 \bar{w}}{\partial \bar{x}^4} = 0 \tag{18}$$

A second-order approximation is sought in the following form

$$\bar{w}(\bar{x}, \bar{t}, \varepsilon) = \bar{w}_0(\bar{x}, T_0, T_1, T_2) + \varepsilon \bar{w}_1(\bar{x}, T_0, T_1, T_2) + \varepsilon^2 \bar{w}_2(\bar{x}, T_0, T_1, T_2) + \cdots \tag{19}$$

where $T_0 = \bar{t}$ is a fast scale characterizing motion while $T_1 = \varepsilon \bar{t}$ and $T_2 = \varepsilon^2 \bar{t}$ are slow scales characterizing the modulation of the amplitudes and phases. Hence,

$$\begin{cases} \dfrac{\partial}{\partial \bar{t}} = \dfrac{\partial}{\partial T_0} + \varepsilon \dfrac{\partial}{\partial T_1} + \varepsilon^2 \dfrac{\partial}{\partial T_2} + \cdots \\ \dfrac{\partial^2}{\partial \bar{t}^2} = \dfrac{\partial^2}{\partial T_0^2} + 2\varepsilon \dfrac{\partial^2}{\partial T_0 \partial T_1} + \varepsilon^2 \left(2\dfrac{\partial^2}{\partial T_0 \partial T_2} + \dfrac{\partial^2}{\partial T_1^2} \right) + \cdots \end{cases} \tag{20}$$

Substituting Eqs. (19) and (20) into (18) and then equating the coefficients of like powers of ε yield

$$\bar{P}\frac{\partial^2 \bar{w}_0}{\partial \bar{x}^2} - \frac{\partial^2 \bar{w}_0}{\partial T_0^2} + \frac{\partial^4 \bar{w}_0}{\partial \bar{x}^4} = 0 \tag{21a}$$

$$\bar{P}\frac{\partial^2 \bar{w}_1}{\partial \bar{x}^2} - \frac{\partial^2 \bar{w}_1}{\partial T_0^2} + \frac{\partial^4 \bar{w}_1}{\partial \bar{x}^4} = 2\frac{\partial^2 \bar{w}_0}{\partial T_0 \partial T_1} + \bar{P}\frac{\partial^4 \bar{w}_0}{\partial \bar{x}^4} - \frac{\partial^4 \bar{w}_0}{\partial T_0^2 \partial \bar{x}^2} \tag{21b}$$

$$\bar{P}\frac{\partial^2 \bar{w}_2}{\partial \bar{x}^2} - \frac{\partial^2 \bar{w}_2}{\partial T_0^2} + \frac{\partial^4 \bar{w}_2}{\partial \bar{x}^4} = 2\frac{\partial^2 \bar{w}_0}{\partial T_0 \partial T_2} + 2\frac{\partial^2 \bar{w}_1}{\partial T_0 \partial T_1} + \frac{\partial^2 \bar{w}_0}{\partial T_1^2} - \frac{\partial^4 \bar{w}_1}{\partial T_0^2 \partial \bar{x}^2} + \bar{P}\frac{\partial^4 w_1}{\partial x^4} - 2\frac{\partial^4 \bar{w}_0}{\partial T_0 \partial T_1 \partial \bar{x}^2} \tag{21c}$$

Firstly, the solution of the above Eq. (21a) can be written as

$$\bar{w}_0(\bar{x},T_0,T_1,T_2)=A(T_1)B(T_2)\,sin\left(n\sqrt{\bar{P}\varepsilon-1}\pi\bar{x}\right)e^{i\sqrt{[\bar{P}n^2\pi^2-(\bar{P}\varepsilon-1)n^4\pi^4](\bar{P}\varepsilon-1)}T_0}+cc \tag{22}$$

where cc stands for the complex conjugate of its left terms. Substituting Eq. (22) into (21b) yields

$$\bar{P}\frac{\partial^2\bar{w}_1}{\partial\bar{x}^2}-\frac{\partial^2\bar{w}_1}{\partial T_0^2}+\frac{\partial^4\bar{w}_1}{\partial\bar{x}^4}=\left[2i\sqrt{[\bar{P}n^2\pi^2-(\bar{P}\varepsilon-1)n^4\pi^4](\bar{P}\varepsilon-1)}\frac{dA}{dT_1}\right. \tag{23}$$

$$\left.+An^6\pi^6(\bar{P}\varepsilon-1)^3\right]B\,sin\left(n\sqrt{\bar{P}\varepsilon-1}\pi\bar{x}\right)e^{i\sqrt{[\bar{P}n^2\pi^2-(\bar{P}\varepsilon-1)n^4\pi^4](\bar{P}\varepsilon-1)}T_0}$$

Eq. (23) has a bounded solution only if the solvability condition holds. The solvability condition demands that the right side of Eq. (23) be orthogonal to every solution of the homogeneous problems. That is

$$\left\langle\left[2i\sqrt{[\bar{P}n^2\pi^2-(\bar{P}\varepsilon-1)n^4\pi^4](\bar{P}\varepsilon-1)}\frac{dA}{dT_1}\right.\right. \tag{24}$$

$$\left.\left.+An^6\pi^6(\bar{P}\varepsilon-1)^3\right]B\,sin\left(n\sqrt{\bar{P}\varepsilon-1}\pi\bar{x}\right),\quad sin\left(n\sqrt{\bar{P}\varepsilon-1}\pi\bar{x}\right)\right\rangle=0$$

where the inner product is defined by

$$\langle g,\ h\rangle=\int_0^1 ghdx \tag{25}$$

which leads to

$$2i\sqrt{[\bar{P}n^2\pi^2-(\bar{P}\varepsilon-1)n^4\pi^4](\bar{P}\varepsilon-1)}\frac{dA}{dT_1}+An^6\pi^6(\bar{P}\varepsilon-1)^3=0 \tag{26}$$

$$A=C_2e^{\frac{in^6\pi^6(\bar{P}\varepsilon-1)^3}{2\sqrt{[\bar{P}n^2\pi^2-(\bar{P}\varepsilon-1)n^4\pi^4](\bar{P}\varepsilon-1)}}T_1} \tag{27}$$

where C_2 is a constant and a particular solution of Eq. (23) that has annihilated the terms generating the secular terms may be

$$\bar{w}_1=0 \tag{28}$$

Substituting Eqs. (22), (27) and (28) into (21c) gives

$$\bar{P}\frac{\partial^2\bar{w}_2}{\partial\bar{x}^2}-\frac{\partial^2\bar{w}_2}{\partial T_0^2}+\frac{\partial^4\bar{w}_2}{\partial\bar{x}^4}=\left[2i\sqrt{[\bar{P}n^2\pi^2-(\bar{P}\varepsilon-1)n^4\pi^4](\bar{P}\varepsilon-1)}\frac{dB}{dT_2}-n^8\pi^8(\bar{P}\varepsilon-1)^4B\right.$$

$$\left.-\frac{n^{10}\pi^{10}(\bar{P}\varepsilon-1)^5}{4[\bar{P}-(\bar{P}\varepsilon-1)n^2\pi^2]}B\right]\times A\,sin\left(n\sqrt{\bar{P}\varepsilon-1}\pi\bar{x}\right)e^{i\sqrt{[\bar{P}n^2\pi^2-(\bar{P}\varepsilon-1)n^4\pi^4](\bar{P}\varepsilon-1)}T_0} \tag{29}$$

Eliminate the secular terms leads to

$$B=C_3e^{-i\frac{4\bar{P}n^8\pi^8(\bar{P}\varepsilon-1)^4-3n^{10}\pi^{10}(\bar{P}\varepsilon-1)^5}{8[\bar{P}-(\bar{P}\varepsilon-1)n^2\pi^2]\sqrt{[\bar{P}n^2\pi^2-(\bar{P}\varepsilon-1)n^4\pi^4](\bar{P}\varepsilon-1)}}T_2} \tag{30}$$

where C_3 is a constant and also a particular solution of Eq. (29) without the secular terms may be

$$\bar{w}_2=0 \tag{31}$$

Now, we obtain the second-order approximation by the method of multiple scales as

$$\bar{w}(\bar{x},\bar{t},\varepsilon) = sin\left(n\sqrt{\bar{P}\varepsilon - 1}\pi\bar{x}\right) \times e^{i\frac{n^5\pi^5(\bar{P}\varepsilon-1)^3[4\bar{P}\varepsilon-4\varepsilon(\bar{P}\varepsilon-1)n^2\pi^2-4\varepsilon^2\bar{P}(\bar{P}\varepsilon-1)n^2\pi^2+3\varepsilon^2n^4\pi^4(\bar{P}\varepsilon-1)^2+8]+8n\pi\bar{P}(\bar{P}\varepsilon-1)[\bar{P}-2n^2\pi^2(\bar{P}\varepsilon-1)]}{8[\bar{P}-(\bar{P}\varepsilon-1)n^2\pi^2]\sqrt{[\bar{P}-(\bar{P}\varepsilon-1)n^2\pi^2](\bar{P}\varepsilon-1)}}\bar{t}} \quad (32)$$

where we suppose $C_2 = C_3 = 1$ as that in Eq. (17). The following Figures **6** and **7** can be found results approach each other for Figures **2** and 6, and Figures **4** and **7**, respectively. For example, a larger non-dimensional tension results in a higher natural frequency from Figure **7**. This is because a larger axial tension corresponds to higher bending stiffness, and higher structural stiffness enhances the natural frequency of micro-rods. Thus, the model used in this paper is proven valid and reasonable. It should be noted that the separation of variables and multiple scales analysis own errors. The structure-preserving method has attracted increasing research interest in recent years [26-30]. Maybe we can use such a method to solve more complex theoretical models in the future.

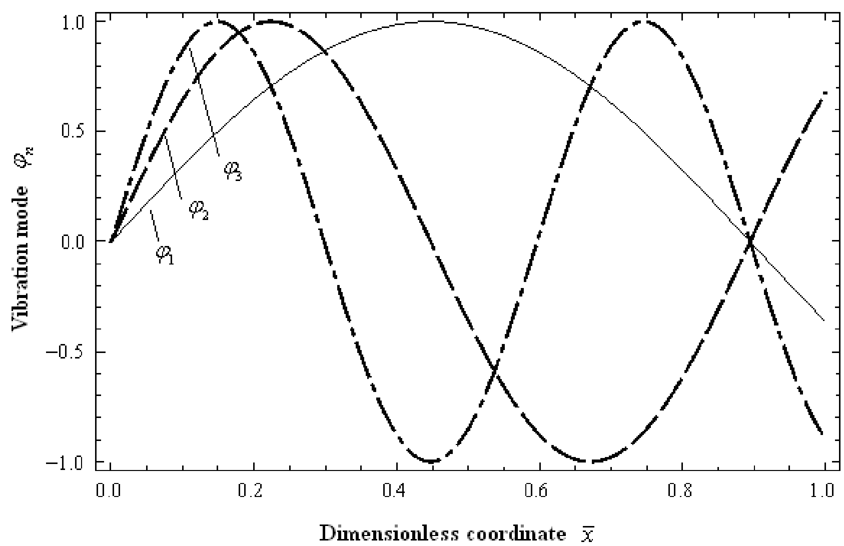

Figure 6: Relations between the first three vibration modes and dimensionless coordinate.

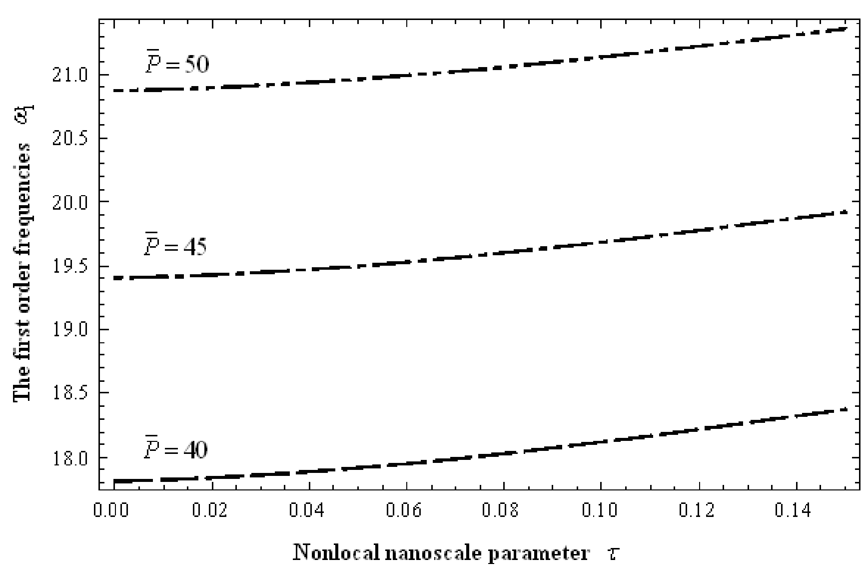

Figure 7: Effects of non-dimensional tension on natural frequencies.

5. Conclusion

As a standard model of microtubules in biological cells and MEMS, the micro-rod model has received more and more research attention in recent years. The present study is devoted to establishing the dynamic model of micro-rods and then solving the theoretical model to obtain some mechanical results at a micro-scale that are different from those at a macro-scale. To this end, the nonlocal theory is used, and the long-range interaction between atoms described by the nonlocal parameter is considered. For the partial differential equation that governs the linear vibration of a simply supported micro-rod, two different methods, including the separation of variables and multiple scales analysis, are applied to determine its vibration characteristics, mainly the natural frequency and vibration mode. We have concluded that the lateral vibration is greatly influenced by the initial axial tension and the nonlocal parameter with a nanoscale, both of which cause the natural frequencies to increase.

Consequently, considering the initial axial tension and the nonlocal effect of the micro-rod increases the equivalent structural stiffness. In designing key components of MEMS, one can increase the initial tensile stress or use materials with higher nonlocal parameters to improve the bending stiffness of the related structures, and both aspects are effective. Results by two different methods are similar, which indicates the correctness of the mathematical modeling built in this paper. The research has a guiding significance for the dynamic design and optimization of related intelligent structures at the micro/nanoscale.

Conflict of Interest

The authors declare no conflicts of interest.

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