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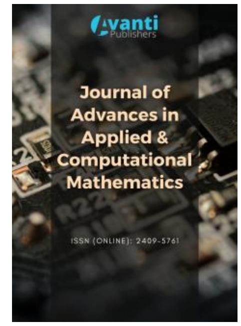

Recursive Estimation in the Moving Window: Efficient Detection of the Distortions in the Grids with Desired Accuracy

Alexander Stotsky*

Department of Computer Science and Engineering, Chalmers University and the University of Gothenburg, SE-412 96 Gothenburg, Sweden

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ABSTRACT

The development of fast convergent and computationally efficient algorithms for monitoring waveform distortions and harmonic emissions will be an important problem in future electrical networks due to the high penetration level of renewable energy systems, smart loads, new types of power electronics, and many others. Estimating the signal quantities in the moving window is the most accurate way of monitoring these distortions. Such estimation is usually associated with significant computational loads, which can be reduced by utilizing the recursion and information matrix properties. Rank two update representation of the information matrix allows the derivation of a new computationally efficient recursive form of the inverse of this matrix and recursive parameter update law. Newton-Schulz and Richardson correction algorithms are introduced in this paper to prevent error propagation and for accuracy maintenance. Extensive comparative analysis is performed on real data for proposed recursive algorithms and the Richardson algorithm with an optimally chosen preconditioner. Recursive algorithms show the best results in estimation with ill-conditioned information matrices.

*Corresponding Author
Email: alexander.stotsky@chalmers.se
Tel: +(46) 707 987504

1. Introduction & Overview of the Paper

The penetration level of grid-connected renewable energy systems, smart loads, and new types of power electronics will increase, and the electric power distribution system will transform based on digital technologies. Power electronics, the key technology for converting electrical power from renewable energy sources to grids, will introduce significant harmonic emissions. These harmonics, together with other distortions, will reduce future electricity networks' reliability, lifetime, and efficiency. Therefore, accurate estimation of the grid events in the presence of significant harmonic emissions, which involves the development of efficient solvers for large-scale algebraic systems (which could be implemented on parallel computational units), is required [1].

Least squares estimation of the frequency contents of the electrical signals in the moving windows is the most accurate way of monitoring network waveform distortions. However, such estimation is usually associated with heavy computational load [2], especially in the case of significant harmonic emissions. The efficiency of this estimation can be improved via the presentation of the information matrix as a recursive rank two updates which takes into account new data which come into the window and the data which leave the window only. The large-scale system of algebraic equations should be solved in each step. The rank two update presentation allows the derivation of the computationally efficient recursive form for the matrix inversion and parameter estimation. The information matrix, which is the sum of rank one matrices (outer products of harmonic regressor) over the moving window, has valuable properties which facilitate parameter calculation. The information matrix is strictly diagonally dominant and positive definite for a sufficiently large window size for systems with harmonic regressors [3]. Moreover, the eigenvalues of this matrix remain the same in all the steps for a fixed window size, which is a new property established in this paper, see Section 2, which allows computationally efficient implementation of the preconditioner based on estimated eigenvalues, see Section 6.1.

The main contribution of this paper is a new recursive form for the inversion of the information matrix with rank two updates in the moving window and recursive parameter calculation algorithms for the least squares problem, see Section 3. Similar to error propagation in the recursive calculations based on rank one update, [4] the error accumulation is one of the main drawbacks of this approach; see Section 4. Error accumulation problems can be solved using Newton-Schulz, [5] and Richardson correction algorithms [6, 7]. These corrections allow recursive parameter estimation with the desired accuracy in the moving window, which makes the overall approach more efficient than existing ones.

Matrix inversion (whose accuracy directly impacts the error propagation) is required at the initial step of the recursive calculations. Small window sizes result in ill-conditioned information matrices and deterioration of the inversion accuracy using existing methods. Therefore, a new method of inversion of the ill-conditioned matrix via a well-conditioned matrix and low-rank update is proposed in this paper; see Section 3. Moreover, the proposed algorithms are compared in this paper with the nonrecursive Richardson algorithm (nonrecursive concerning the parameter vector), which is based on different estimation principles, see Section 6. The Richardson algorithm provides fast and accurate estimation of the parameter vector, can be implemented using matrix-vector multiplications only, and can be easily parallelized. A new method for the determination of the preconditioner for the Richardson algorithm is proposed in this paper. The method is based on truncated Neumann series (similar to the methods described in [8] and [9]) and two scalar preconditioners, see Table **1**. The method allows the determination of the optimal number of the matrix products and matrix-vector multiplications for ill-conditioned information matrices with the best scalar preconditioner. A comprehensive comparative analysis of the proposed recursive algorithms with Richardson algorithms (with the most efficient preconditioner) is performed for different window sizes on real data, which is the second main contribution of this paper, see Section 7. The paper ends with brief conclusions in Section 8, where future research directions are also outlined.

2. Least Squares Estimation in Moving Window

The least squares estimation of the frequency contents of oscillating signal in the window of the size w which is moving in time can be presented in the following form:

$$A_k\theta_k = b_k, \qquad b_k = \sum_{j=k-(w-1)}^{j=k} \phi_j\, y_j = b_{k-1} + d_k \tag{1}$$

$$b_{k-1} = \sum_{j=k-w}^{j=k-1} \phi_j\, y_j\,, \qquad d_k = \phi_k\, y_k - \phi_{k-w}\, y_{k-w} \tag{2}$$

$$A_k = \sum_{j=k-(w-1)}^{j=k} \phi_j\, \phi_j^T = A_{k-1} + R_k \tag{3}$$

$$A_{k-1} = \sum_{j=k-w}^{j=k-1} \phi_j\, \phi_j^T\,, R_k = \phi_k\, \phi_k^T - \phi_{k-w}\, \phi_{k-w}^T \tag{4}$$

$$\phi_k^T = [cos(q_0 k)\; sin(q_0 k) \ldots cos(q_h k)\; sin(q_h k)] \tag{5}$$

where the oscillating signal y_k is approximated using the model $\hat{y}_k = \phi_k^T\theta_k$ with the harmonic regressor (5), where $q_0, \ldots q_h$ are the frequencies. The parameter vector θ_k should be calculated in each step with desired accuracy as the solution of the algebraic equation (1), which is associated with the minimization of the following error $\sum_{j=k-(w-1)}^{j=k} \quad (y_j - \hat{y}_j)^2$, where $y_k = \phi_k^T\theta_*$ and θ_* is the vector of unknown parameters.

The information matrix A_k is defined in (3) as the sum of rank one matrices, and as the rank two updates, R_k of the matrix A_{k-1}, $k \geq w + 1$, Rank two update is associated with the movement of the window, where the new data φ_k , y_k enter the window and the data ϕ_{k-w}, y_{k-w} leave the window in step k.

Notice that the matrix A_k is SPD (Symmetric and Positive Definite) matrix, and the positive eigenvalues of this matrix remain the same in all the steps $k \geq w + 1$ for a given window size w. Evolution of the largest eigenvalue λ_k, for example, such that $A_k v_k = \lambda_k v_k$ can be presented as follows:

$$\begin{aligned}\lambda_k &= v_k^T A_k v_k = v_k^T A_{k-1} v_k + v_k^T R_k v_k \\ &= \lambda_{k-1} + 2v_{k-1}^T A_{k-1}\delta_k + \delta_k^T A_{k-1}\delta_k + v_k^T R_k v_k = \lambda_{k-1}\end{aligned}$$

where $v_k = v_{k-1} + \delta_k$ and δ_k is the increment of the eigenvector due to the rank two update. The evolution of other eigenvalues can be shown using the same arguments.

Another way of evaluating of the eigenvalues of the matrix A_k is a representation of the characteristic polynomial coefficients in terms of traces [10, 11]. For example, the characteristic polynomial of 2×2 information matrix for a single frequency q_0 is $\lambda^2 - \lambda Tr A_k + det\, A_k$, where $det(\, A_k) = \frac{1}{2}[(Tr A_k)^2 - Tr A_k^2]$, where $det\, A_k$ is the determinant and $Tr A_k$ is the trace. Straightforward calculations (using the trigonometric identities presented in [3]) show that $Tr A_k = w$ and

$$Tr A_k^2 = \frac{1}{2}\{\, w^2 + \frac{sin^2(wq_0)}{sin^2(q_0)}\; [cos^2((2k + 1 - w)q_0) + sin^2((2k + 1 - w)q_0)]\,\}$$

This implies that the dependence on the step number k is canceled in the traces (due to the trigonometric identities) and the traces, the coefficients, and the eigenvalues (trace and determinant are sum and product of eigenvalues) depend on the window size w only and remain the same in all the steps. Notice that the determinant equals zero, $det\, A_k = 0\; for\; w = 1$ and the eigenvalues are positive for $w \geq 2$. Eigenvalues of the information matrices for larger frequencies can be evaluated using similar arguments.

3. Recursive Calculation of the Inverse of the Information Matrix and the Parameters

The parameter vector in (1) can be calculated using the inverse of the information matrix $\theta_k = A_k^{-1} b_k$. Denoting $\Gamma_k = A_k^{-1}$ the recursive update of Γ_k via Γ_{k-1} is derived by application of the matrix inversion lemma[1] to the identity (3):

$$\Gamma_k = \Gamma_{k-1} - U_k\, S^{-1}\, U_k^T \tag{6}$$

where $Q_k = [\phi_k\ \phi_{k-w}]$,$U_k = \Gamma_{k-1}\, Q_k$,$S = D + Q_k^T\, \Gamma_{k-1}\, Q_k$, and $D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. The 2×2 matrix S remains the same in all the steps of the window of a given size w (and should be calculated only once), which can be shown using arguments similar to Section 2. Two forms of the parameter update θ_k can be presented as follows:

$$\theta_k = [I - U_k\, S^{-1}\, Q_k^T][\theta_{k-1} + \Gamma_{k-1} d_k] \tag{7}$$

$$\theta_k = \Gamma_k b_k \tag{8}$$

where I is the identity matrix, and the form (7) is derived from (8) and (6). The algorithms are initialized as follows $\Gamma_w = A_w^{-1}$ and $A_w\, \theta_w = b_w$. The parameter update (8) does not depend on the previous step's parameters and only requires matrix-vector multiplication. The inverse matrix and the parameter update law (6) and (7) can be calculated in two parallel loops. Both forms are quadratic complexity algorithms and faster than direct parameter calculation methods.

Inversion of Ill-Conditioned Matrix via Well-Conditioned Matrix and Low-Rank Update. The accuracy of the recursive estimation algorithms depends on the inversion accuracy of the initial information matrix. The initial information matrix can be extremely ill-conditioned for relatively small window sizes, and the inverse may need to be more accurate. The inverse of the ill-conditioned matrix $A_1 = \sum_{j=1}^{w_1} \phi_j \phi_j^T$ can be calculated via the inverse of the matrix with a lower condition number, $A_0 = \sum_{j=1}^{w_0} \varphi_j \varphi_j^T$, $w_0 > w_1$as follows:

$$A_1^{-1} = A_0^{-1} + A_0^{-1}\, Q\, [I - Q^T\, A_0^{-1}\, Q]^{-1}\, Q^T\, A_0^{-1} \tag{9}$$

where the matrix $I - Q^T\, A_0^{-1}\, Q$ with a lower rank and condition number (which can also be relatively high) is inverted only, $Q = [\phi_{w_1+1} \ldots \phi_{w_0}]$.

Notice that inversion with low-rank update can also be divided into several steps and implemented as a recursive procedure (where the matrices of the reduced conditioned numbers are inverted only in each step), which results in the algorithm similar to stepwise splitting [12] and stepwise partitioning, [1]. Recursive procedures usually involve error accumulation and may require corrections [1].

4. Drawbacks of the Recursive Estimation

4.1. Changeable Window Size

Difficulties associated with power and load balancing in electrical networks create challenges in estimating the fast-varying frequency contents of the signals with large fixed window sizes. Inaccuracies in estimation require a window size reduction to capture rapidly changing trends. Reduction of the size of the window results in an ill-

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[1] $(X + YWZ)^{-1} = X^{-1} - X^{-1}Y\, [W^{-1} + Z\, X^{-1}\, Y]^{-1}\, ZX^{-1}$, where $X = A_{k-1},\ Y = Q_k,\ Z = Q_k^T$, and $W = D$

conditioned information matrix, higher computational loads, and delays in estimation. Detection of rapidly and slowly varying trends requires frequently changing window sizes for the best performance in different situations. The algorithms described in Section 3 should be re-initialized when the moving window changes its size. Initialization includes computationally expensive matrix inversion $\Gamma_w = A_w^{-1}$ and the parameter vector calculation, which satisfies $A_w\, \theta_w = b_w$. Notice that the window size adjustment can also be associated with low-rank update, see Section 3, which also requires matrix inversions.

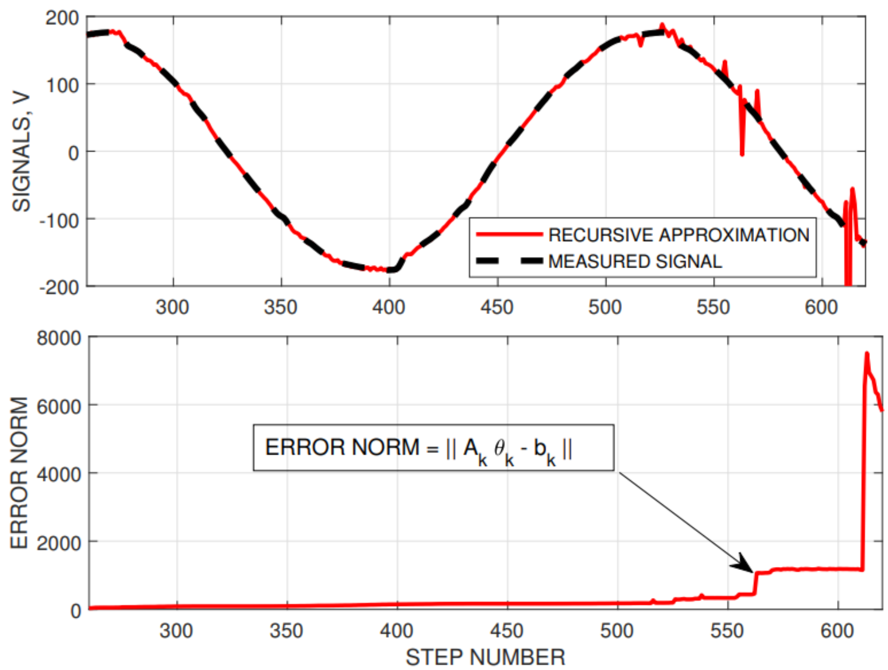

Figure 1: Parameter estimation error $\|A_k\theta_k - b_k\|$ plotted in the second plot for the recursive algorithm (6) and (7) increases with step number due to error accumulation. The measured signal is plotted with the dashed black line, and the recursive approximation in the moving window of the size $w = 254$ is plotted with the solid red line in the first plot.

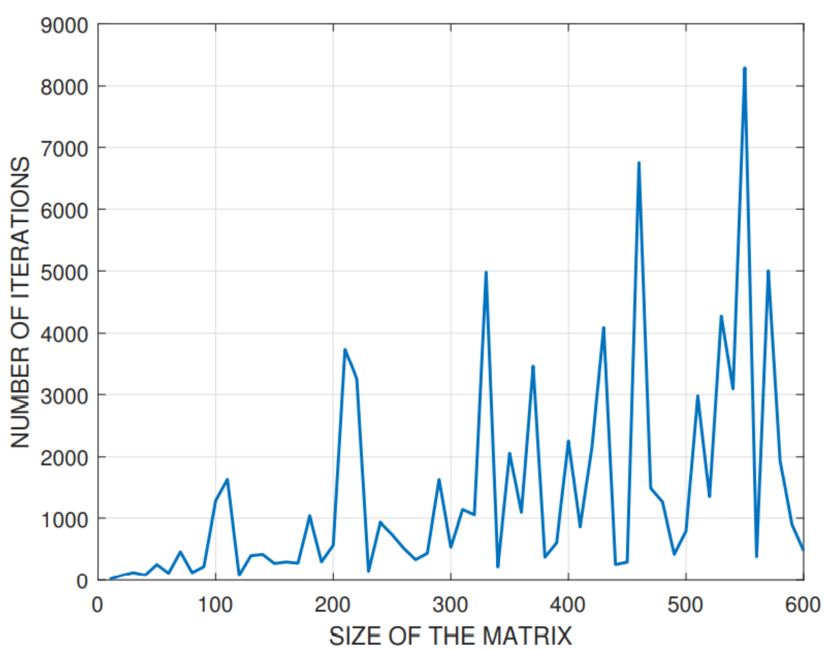

Figure 2: The number of iterations (for estimation of the largest eigenpair $Ax = \lambda x$) as a function of the size of ill-conditioned information matrices required to reach the following accuracy of estimation error: $\|\hat{x}_k\|\ A\,\hat{x}_k\| - A\,\hat{x}_k\| < 0.01$

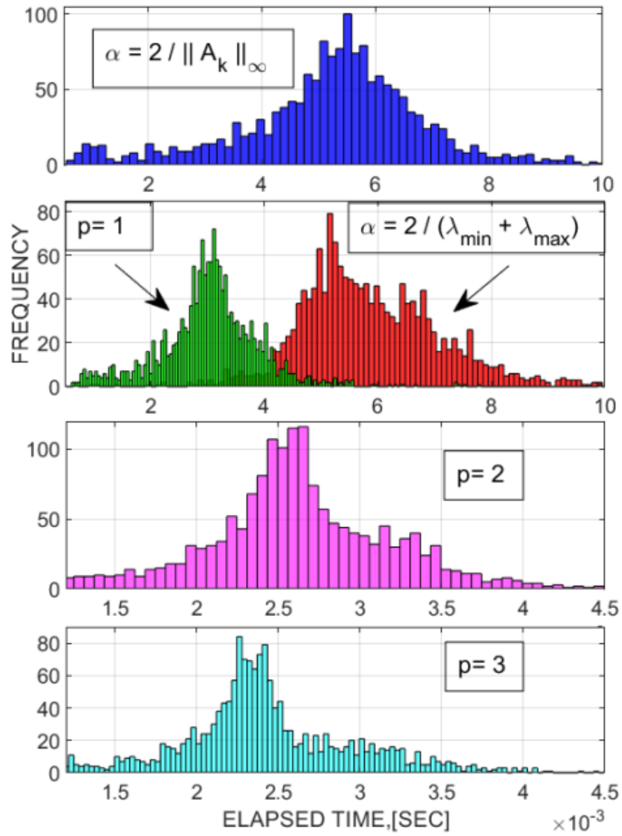

Figure 3: The elapsed time of parameter calculation with the desired accuracy $\|A_k\vartheta_i - b_k\| \le \varepsilon_2 = 0.05$ is measured for five algorithms with different preconditioners. The first and the second plots show the histograms of elapsed time for scalar preconditioners (13) and (14), respectively, where the histogram for (14) is plotted with the red color. The histogram plotted with the green color in the second plot is associated with the first-order preconditioner, $p = 1$ presented in Table **1**. The third and fourth plots are also associated with the preconditioners presented in Table **1**, where the preconditioner of the third order shows the best performance.

The Newton-Schulz method is preferable for calculating the approximate inverse using a few iterations only in the initial step due to corrections applied in the subsequent steps. Notice that the approximate inverse (the inverse with low accuracy) is required only for parameter estimation in the presence of corrections. Direct methods based on Cholesky decomposition, for example, do not allow to choose inversion accuracy and may be less efficient in the initial step, but may require fewer corrections in the subsequent steps.

4.2. Error Accumulation

The algorithm described above can be seen as the ideal explicit recursive solution of the system (1) - (5) in all the steps k. Unfortunately, such a solution could be more robust concerning error accumulation in finite digit calculations. The accumulation strength depends on the size of the moving window and the level of precision. The performance deterioration due to error accumulation is significant for ill-conditioned information matrices. Fortunately, it is not significant for relatively large window sizes but may be pronounced for big data applications.

The effect of error accumulation for the recursive algorithm (6) and (7) is illustrated in Fig. (**1**), where the parameter estimation error $\|A_k\theta_k - b_k\|$ is plotted in the second plot and increases with the step number. Error accumulation has a direct impact on the approximation performance shown in the first plot of Fig. (**1**), where the measured signal (the one-phase voltage waveform measured at the wall outlet) is plotted with the dashed black line and recursive approximation (with 120 harmonics) in the moving window of small size, $w = 254$ is plotted with the solid red line. The measurement data was provided by the IEEE Working Group on Power Quality Data Analytics [13]. The corrections for the elimination of error accumulation are presented in the following Section 5.

5. Estimation with Desired Accuracy and Recursive Preconditioning

Newton-Schulz matrix inversion and Richardson algorithms can be applied for corrections [7]. The inversion error $F_i = I - G_i A_k$ and the parameter estimation error $E_i = A_k \vartheta_i - b_k$ drive the following corrections:

$$\text{Update } G_i = \sum_{j=0}^{n} F_i^j G_{i-1} \text{ while } \|F_i\| > \varepsilon_1, G_0 = \Gamma_k \tag{10}$$

$$\text{Update } \vartheta_i = \vartheta_{i-1} - G_i E_{i-1} \text{ while } \|E_i\| > \varepsilon_2, \vartheta_0 = \theta_k \tag{11}$$

Furthermore, activated in cases where errors exceed pre-specified bounds, $\varepsilon_{1,2} > 0$. Low order (for example $n = 2$) should be selected for the Newton-Schulz matrix inversion algorithm. The matrix Γ_k (for which the spectral radius $\rho(I - \Gamma_k A_k) << 1$ is much less than one) and θ_k play the role of efficiently calculated preconditioner and a priori parameter estimate, respectively. Newton-Schulz matrix inversion and Richardson algorithms are ideally suited for these corrections providing (after a few iterations only, i=1,2,..., see Section 7) two improved estimates (G_i for Γ_k and ϑ_i for θ_k) for the next step of the recursion.

6. Nonrecursive Richardson Algorithm

The Nonrecursive Richardson algorithm described, for example, in [6] and [14], which requires matrix-vector multiplications, can be used directly for the calculation of the parameters θ_k in (1)

$$\theta_i = \theta_0 - \sum_{j=0}^{i_*} F_0^j \; G_0 \; (A_k \theta_0 - b_k), \qquad F_0 = I - G_0 A_k \tag{12}$$

via power series expansion until the accuracy requirement is fulfilled. The performance of the algorithm (12) depends on the initial values θ_0 and G_0. Examination of the terms in (7) shows that the matrix $I - U_k S^{-1} Q_k^T$ is relatively close to the identity matrix, and the contribution of the term $\Gamma_{k-1} d_k$ is relatively small. Therefore the initial value can be taken as $\theta_0 = \theta_{k-1}$ or $\theta_0 = G_0 b_k$ where G_0 is sufficiently close to A_k^{-1}.

6.1. Preconditioning Based on the Properties of the Window

The matrix G_0 in (12) can be chosen as $G_0 = \alpha I$ where the scalar preconditioner (13)

$$\alpha = \frac{2}{\|A_k\|_\infty} \tag{13}$$

$$\alpha = \frac{2}{\hat{\lambda}_{min}(A_k) + \hat{\lambda}_{max}(A_k)} \tag{14}$$

guarantees that the spectral radius ρ of the SPD matrix A_k is less than one, $\rho(I - \alpha A_k) < 1$ where $||\cdot||_\infty$ is the maximum row sum matrix norm [7, 15].

The spectral radius of the matrix $(I - \alpha A_k)$ gets its minimal value $(1 - \hat{\lambda}_{min}(A_k)\alpha)$ for the SPD matrix A_k for the preconditioner (14), where $\hat{\lambda}_{min}(A_k)$ and $\hat{\lambda}_{max}(A_k)$ are the estimates of minimal and maximal eigenvalues of A_k, respectively. In other words, the preconditioner (14) maps the interval containing all eigenvalues of A_k onto symmetric interval around the origin [15].

The following power (Von Mises) iteration algorithm [16], which requires only matrix-vector multiplications $\hat{x}_k = \frac{A\,\hat{x}_{k-1}}{||A\,\hat{x}_{k-1}||}$, can be applied to estimate the largest eigenpair $A\,x = \lambda_{max}(A)\,x$.

Notice that the minimal eigenvalue of A can be estimated via the maximal eigenvalue of $(\beta I - A)$ where $\beta = \hat{\lambda}_{max}(A) + \varepsilon$, and ε is a sufficiently small positive number. The maximal eigenvalue $(\beta I - A)$, in turn, can be estimated using the same algorithm.

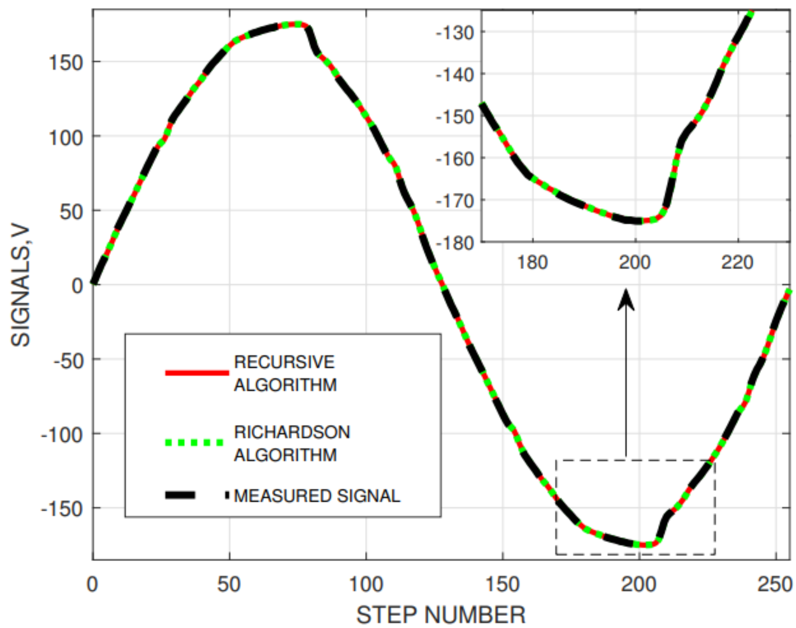

Figure 4: The Figure shows approximation performance (in the moving window of the size $w = 254$ with the desired parameter estimation accuracy $\varepsilon_2 = 0.05$) of the recursive algorithm (6), (8) plotted with a red line, and the algorithm (12) plotted with the green dotted line. The measured signal is plotted with the dashed black line.

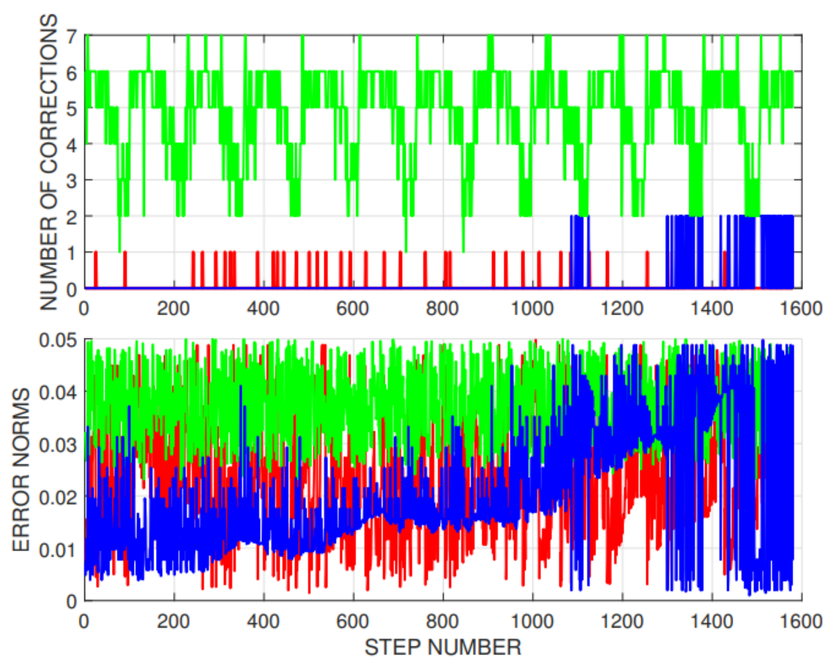

Figure 5: The Figure shows the number of corrections to achieve the desired parameter estimation accuracy $\|A_k\vartheta_i - b_k\| \leq \varepsilon_2 = 0.05$ in the first plot and the error norms $\|A_k\vartheta_i - b_k\|$ (in the second plot) for the algorithms (7), (8), and the algorithm (12), which are plotted with red, blue and green colors respectively in both plots for the window size $w = 310$.

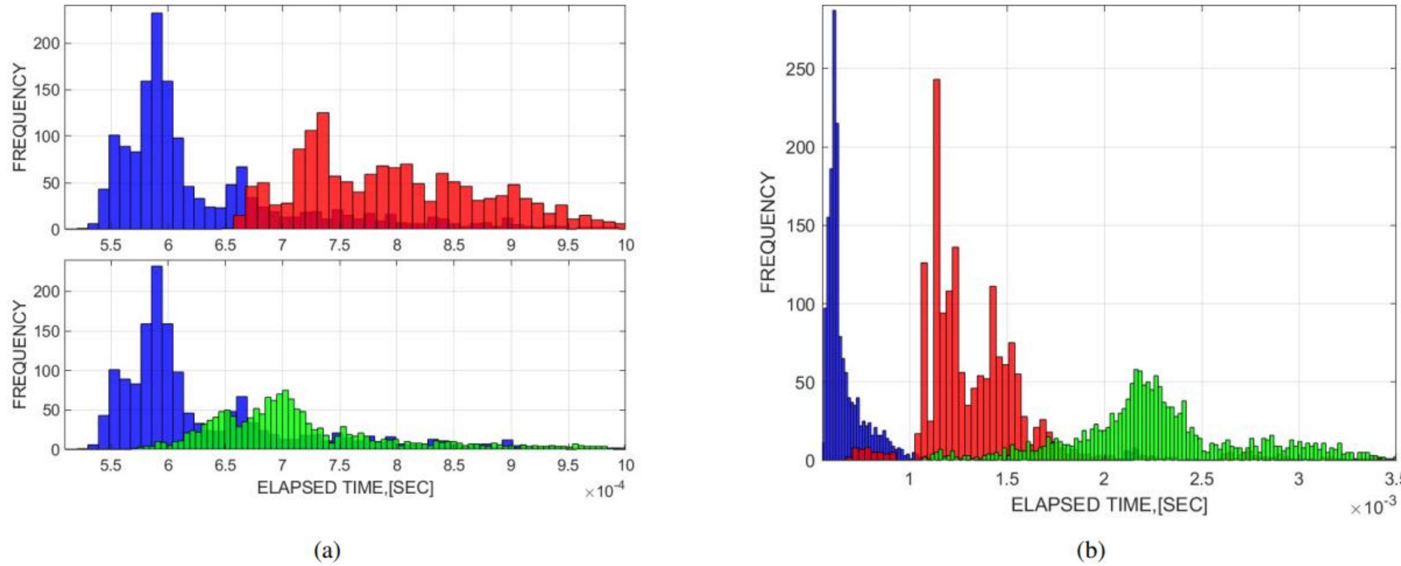

Figure 6: The elapsed time of parameter calculation with the desired accuracy $||A_k\vartheta_i - b_k|| \leq \varepsilon_2 = 0.05$ is measured for three algorithms for different sizes of moving windows. Fig. (**6a**) corresponds to window size $w = 310$ and Fig. (**6b**) corresponds to $w = 254$. The histograms for algorithm (8) are plotted with blue color, and histograms for algorithm (7) and algorithm (12) are plotted with red and green colors, respectively.

Even though estimating the eigenvalues in the moving window should be performed only once (since the eigenvalues remain the same for a fixed window size, see Section 2), the number of iterations can be sufficiently large for a large size of the ill-conditioned information matrices. The number of iterations as a function of the size of the ill-conditioned information matrices that is required to reach the following accuracy of estimation error: $\|\hat{x}_k\|\ A\ \hat{x}_k\|-A\ \hat{x}_k\| < 0.01$ is plotted in Fig. (**2**). The Fig. (**2**) shows that the number of iterations increases with the size of the matrix and can be sufficiently large (which corresponds to a large number of matrix-vector multiplications) for large scale systems. Notice that the power iteration algorithm can be easily parallelized, which essentially improves the efficiency of this preconditioner.

6.2. Neumann Series-Based Preconditioners and Comparison

Initial error in (12) can be reduced via the application of the power series expansion with scalar preconditioners (13) and (14); see Table **1**.

Table 1: Preconditioners based on truncated Neumann series.

Order p	$G_0 = [\sum_{j=0}^{p} F_0^j]\alpha, F_0 = I - \alpha A_k, I - G_0 A_k = F_0^{p+1}$ $\alpha = \frac{2}{\|\|A_k\|\|_\infty}$ or $\alpha = \frac{2}{\bar{\lambda}_{min}(A_k)+\bar{\lambda}_{max}(A_k)}$, $\rho(F_0) < 1$
1	$G_0 = 2\alpha - \alpha^2 A_k$
2	$G_0 = 3\alpha - 3\alpha^2 A_k + \alpha^3 A_k^2$
3	$G_0 = 4\alpha - 6\alpha^2 A_k + 4\alpha^3 A_k^2 - \alpha^4 A_k^3$
…	…

The one-phase voltage waveform measured at the wall outlet (approximately 120V RMS) [13] compares different preconditioners. The sampling measurement rate is 256 points per cycle. The signal was approximated with the system with harmonic regressor (5), which contains 120 harmonics with the fundamental frequency of $q_0 = 60$ Hz. Short window size $w = 254$ was selected in order to find the best preconditioner for ill-conditioned information matrices. A comparison of the performance of different preconditioners is presented in Fig. (**3**), where histograms of the elapsed time of parameter calculation with the desired accuracy $\|A_k \vartheta_i - b_k\| \leq \varepsilon_2 = 0.05$ are measured for five algorithms with different preconditioners. The first and the second plots show the histograms of elapsed time for scalar preconditioners (13) and (14), respectively. The preconditioner (14) shows no significant improvements concerning (13). Besides, estimating the minimal and maximal eigenvalues requires significant computational efforts; see Fig. (**2**) every time the window size changes. Therefore, the preconditioner (14) (without parallelization) is not recommended for this application. The first-order preconditioner in Table **1**, $p = 1$ whose histogram is plotted in the second plot with the green color, shows a significant reduction of the computational time compared to scalar preconditioners.

The third and fourth plots in Fig. (**3**) are associated with the preconditioners presented in Table **1**, calculated with a scalar preconditioner (13). The third-order preconditioner shows the best performance, and further increase of the order of the power series in Table **1** does not show any improvements but shows deterioration of the performance.

7. Comparison of Recursive and Nonrecursive Estimation on Real Data

The same one-phase voltage waveform measured at the wall outlet is used for comparisons; see Section 6.2. Three algorithms are compared: 1) the parameter estimation algorithm (7); 2) the parameter estimation algorithm (8) (both with the recursive estimate of the inverse (6)) ; 3) the nonrecursive Richardson algorithm (12) with the best preconditioner chosen in Section 6.2.

Estimation is performed with the desired accuracy of inversion of the information matrix $\varepsilon_1 = 0.1$ and the parameter estimation accuracy $\varepsilon_2 = 0.05$ (for the third algorithm, the parameter accuracy is relevant only). The simulations were performed in Matlab. Approximation performance, see Fig. (**4**); the number of corrections, see Fig. (**5**) and the elapsed time, see Fig. (**6**) of the parameter calculation are evaluated for three algorithms for different sizes of the moving window, ($w = 310$ and $w = 254$) which has an impact on the condition number (the condition number increases with the reduction of the window size).

Fig. (**4**) shows outstanding approximation performance with 120 harmonics for all the algorithms, and the second plot in Fig. (**5**) shows the fulfillment of the accuracy requirements, $\varepsilon_2 = 0.05$. The number of corrections for the achievement of the desired accuracy and error norms which are plotted in Fig. (**5**), shows that algorithm (7) is more accurate than an algorithm (8) and requires a smaller number of corrections. However, calculation of the parameter vector θ_k via θ_{k-1} (7) is more expensive than one or two steps of corrections which makes algorithm (8) faster than (7) in estimation with the desired accuracy, see Fig. (**6a**). Richardson algorithm (12) requires few iterations only for $w = 310$ with estimation accuracy close to $\varepsilon_2 = 0.05$, see the second plot in Fig. (**5**). It is more efficient than the algorithm (7), see Fig. (**6a**). For smaller window sizes the number of steps of the Richardson algorithm (12) increases, see Fig. (**6b**) and the algorithms (7) and (8) have better performance.

Notice that the family of Richardson algorithms provides faster parameter calculations for lower levels of accuracy (in approximate computing), does not require matrix inversion if window size changes and can be easily implemented on parallel computational units. New recursive forms (7) and (8) and Richardson algorithms have different application areas. The algorithms (7) and (8) with corrections can be recommended for highly ill-conditioned cases where the window size does not change often. Richardson algorithms can be easily parallelized and applied for frequently varying window sizes with lower accuracy requirements.

8. Conclusion

This paper proposes new computationally efficient recursive forms with corrections for parameter estimation in moving windows. The algorithms utilize the computational efficiency of the recursive forms based on the matrix inversion lemma and the advantages of Newton-Schulz and Richardson corrections, which allow the elimination of the error accumulation and monitoring of the grid events with the desired accuracy. Extensive simulations on real data showed that new algorithms are incredibly efficient for monitoring rapidly changing trends in short windows and ill-conditioned information matrices.

Moving windows for systems with harmonic regressors have exciting properties that should be further studied and utilized for designing and implementing computationally efficient and accurate monitoring algorithms in future electrical networks.

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