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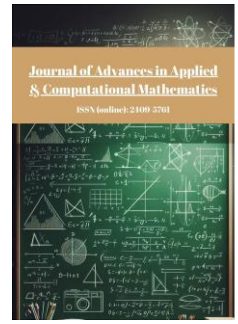


# A Study of an EOQ Model of Deteriorated Items with Pentagonal Dense Fuzzy Demand Rate

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## ABSTRACT

In this project work, we deal with an economic order quantity inventory model of deteriorating items under non-random uncertain demand. Here we consider the customers screen the fresh items during the selling period. After a certain period of time, the deteriorated items are sold at a discounted price. Firstly, we solve the crisp model, and then the model is converted into a fuzzy environment. Here we consider the pentagonal dense fuzzy, trapezoidal dense fuzzy, and triangular dense fuzzy for a comparative study. We have taken the numerical result using LINGO 18.0 software. Finally, sensitivity analysis and graphical illustration have been given to check the validity of the model.

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# 1. Introduction

Zadeh [1] created fuzzy sets as an extension of classical set theory that addresses the notion of partial membership. Fuzzy sets, as indicated by a membership function ranging from 0 to 1, allow elements to belong to a set to variable degrees, in contrast to standard binary sets where elements are either in or out. Fuzzy sets are especially helpful in modeling imprecise or unclear information, which is frequently seen in real-world circumstances due to their flexibility. As a result, they find use in many different domains, including artificial intelligence, decision-making, and control systems. Chang and Zadeh [2] introduced the concept of fuzzy mapping and control. Various types of fuzzy numbers, such as L-R fuzzy number, triangular fuzzy number, trapezoidal fuzzy number, pentagonal fuzzy number, hexagonal fuzzy number, heptagonal fuzzy number, half-circle fuzzy number, exponential fuzzy number, and Gaussian fuzzy number, are widely recognized as general fuzzy numbers. However, the insertion of learning experiences can modify these fuzzy numbers into other variations. Researchers like, De and Mahata [3] (cloudy fuzzy number), De and Beg [4, 5] (dense fuzzy number), De [6] (triangular dense fuzzy lock set), among others, have delved into these directions.

However, none of these methods have yet provided an entirely effective defuzzification output based on learning experiences. Therefore, in this paper, we explore various types of fuzzy numbers under dense fuzzy rules, which are closely linked with learning experiences in decision-making processes.

In contemporary research, scholars are enhancing traditional backorder Economic Order Quantity (EOQ) models through various approximations and methodologies involving uncertain parameters. A plethora of scholarly articles in the literature delve into characterizing inventory problems. For instance, De and Sana [7] devised a backlogging EOQ model under intuitionistic fuzzy set (IFS) framework, employing the score function of the objective value. De *et al*. [8] applied the IFS technique by interpolation to formulate a backorder EOQ model. Meanwhile, in an IFS environment, De [9] explored a specialized EOQ model incorporating natural idle time (the general closing time duration per day).

Das *et al*. [10] introduced a step-order fuzzy model to address time-dependent backlogging during idle time. Simultaneously, De and Sana [11] crafted an alternative fuzzy EOQ model with backlogging, sensitive to selling price and promotional effort in demand. Kazemi *et al*. [12] integrated the learning effect on fuzzy parameters into an EOQ model for imperfect quality items, also considering human forgetting effects.

Several fuzzy inventory models have been accumulated by Shekarian *et al*. [13], providing a comprehensive review. Imperfect quality items were further analyzed by Shekarian *et al*. [14] and Patro *et al*. [15]. Additionally, other contemporary researchers (Shekarian *et al*. [16], Sharma and Govindaluri [17], Chanda and Kumar [18]) have enriched the fuzzy inventory model landscape. Karmakar *et al*. [19] explored a pollution-sensitive production inventory model utilizing dense fuzzy numbers and a new defuzzification method intelligently. Karmakar *et al*. [20] developed another pollution-sensitive remanufacturing model using triangular dense fuzzy lock sets, marking a significant milestone in the related field. According to the latest literature survey, Maity *et al*. [21] investigated articles on learning experiences, presenting an EOQ model for decision-making involving two decision makers, utilizing triangular dense fuzzy lock sets effectively. However, Maity [22] solved an inventory problem under intuitionistic fuzzy environment. Maity *et al*. [23] proposed an EOQ model for imperfect items, where customers evaluate items prior to purchase. Furthermore, Maity *et al*. [24] defined arithmetic operations over parabolic dense fuzzy lock sets, applying them to resolve inventory dilemmas. Additionally, Maity *et al*. [25] examined an EOQ model under uncertain daytime demand rates, offering a computer-based algorithm and flowchart for model optimization. Lastly, Maity *et al*. [26] defined a cloud-type non-linear intuitionistic dense fuzzy set, considering both symmetry and asymmetry cases. Moreover, an economic production quantity (EPQ) model with deterioration was developed by Rahaman *et al*. [27]. In this model, the demand rate is unit selling price and stock dependent, whereas the production rate is stock dependent. However, a three-echelon supply chain model with a single manufacturer, supplier, and distributor under fuzzy system was described by Mahata *et al*. [28]. Mallik and Maity [29] investigated an inventory model under cloudy fuzzy environment. Recently, Maity *et al*. [30] studied over a green inventory model of degrading products.

The aforementioned research reveals a gap in the utilization of pentagonal dense fuzzy environments among scholars. Therefore, this study aims to fill this gap by presenting an EOQ model for deteriorated items within a pentagonal dense fuzzy framework. A comparative analysis is provided to justify the efficacy of our proposed model.

The remaining part of this article is organized as follows: Some basic definitions of fuzzy set and corresponding defuzzification method are mentioned in section 2. The crisp mathematical model has been formulated in section 3. Section 4 contains the corresponding fuzzy model. The numerical result of our proposed model has been given in section 5. Graphical illustration and the conclusion are provided in sections 6 and 7 respectively.

# 2. Preliminaries

In this chapter, we introduce different definitions of dense fuzzy sets and triangular dense fuzzy set with their graphic representations and examples for subsequent use (Fig. **1**).

**Definition 1:** Consider the fuzzy set $\tilde{A}$ whose components are a sequence of functions generated from the mapping of natural numbers with a crisp number *x*. Now if all the components converge to the crisp number *x* as *n* → ∞ then the fuzzy sets under consideration are called dense fuzzy set (DFS).

$$\left(\left(\left(\left(\left(\left(((x))\right)\right)\right)\right)\right)\right) \to\to\to\to\to (x) \to\to x$$

**Figure 1:** Dense fuzzy set.

**Definition 2:** Let a fuzzy number $\tilde{A} = < a_1, a_2, a_3 >$ with $a_1 = a_2 f_n$ and $a_3 = a_2 g_n$, where $f_n$ and $g_n$ are the sequence of functions. If $f_n$ and $g_n$ are both converges to 1 as $n \to \infty$ then the fuzzy set $\tilde{A} = < a_1, a_2, a_3 >$ converges to a crisp singleton $\{a_2\}$. Then we call the fuzzy set $\tilde{A}$ = $< a_1, a_2, a_3 >$ as a Triangular dense fuzzy set (TDFS).

**Definition 3:** Definition of TDFS based on Cartesian product of two sets. Let $\tilde{A}$ be the fuzzy number whose components are the elements of $R \times N, R$ being the set of real numbers and *N* being the set of natural numbers with the membership grade satisfying the functional relation $\mu: R \times N \to [0,1]$. Now as $n \to \infty$if $\mu(x,n) \to 1$ for some $x \in Randn \in N$ then we call the set $\tilde{A}$ as dense fuzzy set. If $\tilde{A}$ is triangular then it is called TDFS. Now, if for some $n$ in $N, \mu(x,n)$ attains the highest membership degree 1 then the set itself is called Normalized Triangular Dense Fuzzy Set (NTDFS).

**Definition 4:** The set $\tilde{P} = (p_1, p_2, p_3, p_4, p_5; \gamma_{\tilde{P}})$ is called a pentagonal fuzzy number (PFN) if the MF $\gamma_{\tilde{P}}(x)$ satisfies the following conditions,

(1) $\gamma_{\tilde{P}}(x)$ is continuous and lies within [0,1]

(2) $\gamma_{\tilde{P}}(x)$ is strictly increasing within $[m_1, m_2]$ and $[m_2, m_3]$

(3) $\gamma_{\tilde{P}}(x)$ is strictly decreasing within $[m_3, m_4]$ and $[m_4, m_5]$

**Definition 5:** A pentagonal fuzzy number with asymmetry (PFNA) $\left(\widetilde{A_{PIFNA}}\right)$ is defined as$\widetilde{A_{PIFNA}} = \langle [(e_1, e_2, e_3, e_4, e_5 | t, s); \varphi] \rangle$, $\varphi \in [0,1]$. The MF $\varphi_{\widetilde{A_{PIFNA}}}: \mathbb{R} \to [0,1]$ is characterized by:

$$\varphi_{\widetilde{A_{PIFNA}}}(x) = \begin{cases} t\dfrac{x - e_1}{e_2 - e_1} \ \ if \ e_1 \leq x \leq e_2 \\ t + (1-t)\dfrac{x - e_2}{e_3 - e_2} \ \ if \ e_2 \leq x \leq e_3 \\ 1 \ \ if \ x = e_3 \\ s + (1-s)\dfrac{e_4 - x}{e_4 - e_3} \ if \ e_3 \leq x \leq e_4 \\ s\dfrac{e_5 - x}{e_5 - e_4} \ \ if \ e_4 \leq x \leq e_5 \\ 0 \ \ \ if \ x > e_5 \end{cases}$$

**Definition 6:** A pentagonal dense fuzzy number with asymmetry (PDFNA) $\left(\widetilde{A_{PIDFNA}}\right)$ is characterized as $\widetilde{A_{PIDFNA}} = \langle k_1\left(1-\frac{\theta_1}{1+n}\right), k_1\left(1-\frac{\theta_2}{1+n}\right), k_1, k_1\left(1+\frac{\varphi_1}{1+n}\right),\ k_1\left(1+\frac{\varphi_2}{1+n}\right), m, p, r, s\rangle$, in which MF $\mu_{\widetilde{A_{PIDFNA}}}(x)$ is specified as:

$$\mu_{\widetilde{A_{PIDFNA}}}(x) = \begin{cases} m\left\{\dfrac{x-\ k_1\left(1-\frac{\theta_1}{1+n}\right)}{\frac{k_1(\theta_1-\theta_2)}{1+n}}\right\} & if\ \ k_1\left(1-\dfrac{\theta_1}{1+n}\right) \le x \le k_1\left(1-\dfrac{\theta_2}{1+n}\right) \\ m+(1-m)\left\{\dfrac{x-\ k_1\left(1-\frac{\theta_2}{1+n}\right)}{\frac{k_1\theta_2}{1+n}}\right\} & if\ \ k_1\left(1-\dfrac{\theta_2}{1+n}\right) \le x \le\ k_1 \\ p+(1-p)\left\{\dfrac{k_1\left(1+\frac{\varphi_1}{1+n}\right)-x}{\frac{k_1\varphi_1}{1+n}}\right\} & if\ \ k_1 \le x \le\ k_1\left(1+\dfrac{\varphi_1}{1+n}\right) \\ p\left\{\dfrac{k_1\left(1+\frac{\varphi_2}{1+n}\right)-x}{\frac{k_1(\varphi_2-\varphi_1)}{1+n}}\right\} & if\ \ k_1\left(1+\dfrac{\varphi_1}{1+n}\right) \le x \le k_1\left(1+\dfrac{\varphi_2}{1+n}\right) \end{cases}$$

The graph of membership function of PDFNA is given in Fig. (**2**) (Maity *et al*. [30]).

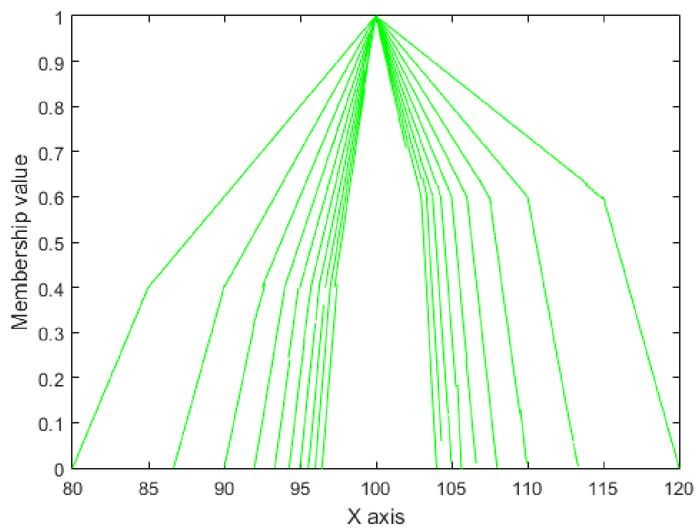


**Figure 2:** Membership function of PDFNA.

### 2.1. Defuzzification Formula of PDFN

Let $\tilde{A} = \langle a_1, a_2, a_3, a_4, a_5\rangle$ be a PIDFN with membership function $\mu(x) \in [0,1]$. Then the defuzzification formula of $\tilde{A}$ is given by

$$\mathrm{I}(\tilde{A}) = \frac{\int_{a_1}^{a_2} x\,\mu(x)dx+\int_{a_2}^{a_3} x\,\mu(x)dx+\int_{a_3}^{a_4} x\,\mu(x)dx+\int_{a_4}^{a_5} x\ \ \mu(x)dx}{\int_{a_1}^{a_2} \mu(x)dx+\int_{a_2}^{a_3} \mu(x)dx+\int_{a_3}^{a_4} \mu(x)dx+\int_{a_4}^{a_5} \mu(x)dx}$$

# 3. Crisp Mathematical Model

The notations and assumptions of our proposed model are given below.

### 3.1. Notation

$h_c$: Holding cost per unit item per unit time ($)

$o_c$ : Ordering cost per order ($)

$s_r$ : Supply rate of the items (unit)

$d$: Demand rate during time $t = 0 \; to \; t = t_2$

$d_1$ :Demand rate in the discount period (Unit)

$c_p$ : Purchasing price per unit item ($)

$s_p$ : Selling price per unit item ($)

$J_1$ : Inventory level at time $[0, t_1]$

$J_2$: Inventory level at time $[t_1, t_2]$

$J_3$ : Inventory level at time $[t_2, T]$

$\theta_1$ : Deterioration rate in [0,$t_1$]

$\theta_2$ : Deterioration rate in [$t_1$,T]

$s_c$ : Salvage cost of deteriorated item ($/unit)

$\eta$ : Salvage coefficient $(0 \leq \eta \leq 1)$

### 3.2. Assumption

We have made the following assumptions for our proposed model.

i. Demand rate is uniform and known.

ii. Rate of replenishment is finite

iii. Shortages are not allowed

iv. Supply rate is finite ($s_r$ units per unit time)

v. Lead time is zero

### 3.3. Formulation of Crisp Mathematical Model

$$\frac{dJ_{1\,(t)}}{dt} = s_r - d - \theta_1 J_1(t) \text{for } 0 \leq t \leq t_1 \tag{1}$$

$$\frac{dJ_{2(t)}}{dt} = -d - \theta_2 J_2(t) \text{ for } t_1 \leq t \leq t_2 \tag{2}$$

$$\frac{dJ_{3(t)}}{dt} = -d_1 - \theta_2 J_3(t) \text{ for } t_2 \leq t \leq T \tag{3}$$

with boundary condition $J_1(0) = J_3(T) = 0$

and $J_1(t_1) = J_2(t_1), J_2(t_2) = J_3(t_2)$

$$\frac{dJ_1\,(t)}{dt} + \theta_1 J_1\,(t) = s_r - d$$

After solving the above equation, we get

$$J_1(t) = \frac{s_r - d}{\theta_1} - \frac{s_r - d}{\theta_1} e^{-\theta_1 t} \tag{4}$$

$$= \frac{s_r - d}{\theta_1}\left[1 - e^{-\theta_1 t}\right]$$

$$\frac{dJ_2\;(t)}{dt} + \theta_2 J_2\,(\text{t}) = -\text{d}$$

$\Rightarrow J_2\,(\mathrm{t}) \; = -\frac{d}{\theta_2} + c_2 e^{-\theta_2 t}$ (5)

$\frac{dJ_3(t)}{dt} + \theta_2 J_3(t) = -d_1$

$\Rightarrow \; J_3(t) = -\frac{d_1}{\theta_2} + \frac{d_1}{\theta_2} e^{\theta_2\,(T-t)}$ (6)

$= \frac{d_1}{\theta_2}[e^{\theta_2(T-t)} - 1]$

$J_1(t_1) = J_2(t_1)\,, J_2(t_2) = J_3(t_2)$

For t $= t_2$

$-\frac{d}{\theta_2} + c_2 e^{-\theta_2 t_2} = \frac{d_1}{\theta_2} e^{-\theta_2(T-t_2)} - \frac{d_1}{\theta_2}$

$c_2 e^{-\theta_2 t_2} = \frac{d_1}{\theta_2} e^{\theta_2(T-t_2)} - \frac{d_1}{\theta_2} + \frac{d}{\theta_2}$

$c_2 = \frac{d_1}{\theta_2} e^{\theta_2 T} + (\frac{d}{\theta_2} - \frac{d_1}{\theta_2}) e^{-\theta_2 t_2}$

$\therefore J_2(t) = -\frac{d}{\theta_2} + [\frac{d_1}{\theta_2} e^{\theta_2 T} + \left(\frac{d}{\theta_2} - \frac{d_1}{\theta_2}\right) e^{-\theta_2 t_2}] e^{-\theta_2 t}$

Holding cost

$= h_c \left\{\int_0^{t_1} J_1(t)dt + \int_{t_1}^{t_2} J_2(t)dt + \int_{t_2}^{T} J_3(t)dt\right\}$

$= h_c \left\{\frac{s_r - d}{\theta_1}\left[t_1 + \frac{e^{-\theta_1 t_1}}{\theta_1} - \frac{1}{\theta_1}\right] - \frac{d}{\theta_2}(t_2 - t_1) - \frac{d_1}{\theta_2^2}\left[e^{\theta_2(T-t_2)} - e^{\theta_2(T-t_1)}\right] - \frac{d-d_1}{\theta_2^2}\left[e^{-2t_2\theta_2} - e^{-\theta_2(t_2+t_1)}\right] + \frac{d_1}{\theta_2}\left[-\frac{1}{\theta_2} + \frac{1}{\theta_2} e^{\theta_2(T-t_2)} - (T - t_2)\right]\right\}$

Deterioration cost per cycle

$= c_p\{s_r t_1 - \int_0^{t_2} d\,dt - \int_{t_2}^{T} d_1 dt\}$

$= c_p\{s_r t_1 - d\; t_2 - d_1(T - t_2)\}$

Salvage value of deteriorated items per cycle

$= s_c \eta\{s_r t_1 - \int_0^{t_2} d\,dt - \int_{t_2}^{T} d_1 dt\}$

$= s_c\,\eta\{s_r t_1 - dt_2 - d_1\,(\mathrm{T} - t_2)\}$

Total cost

=Holding cost+Deterioration cost+Purchasing cost+ Ordering cost

$= h_c \left\{\frac{s_r - d}{\theta_1}\left[t_1 + \frac{e^{-\theta_1 t_1}}{\theta_1} - \frac{1}{\theta_1}\right] - \frac{d}{\theta_2}(t_2 - t_1) - \frac{d_1}{\theta_2^2}\left[\theta e^{\theta_2(T-t_2)} - e^{\theta_2(T-t_1)}\right] - \frac{d-d_1}{\theta_2^2}\left[e^{-2t_2\theta_2} - e^{-\theta_2(t_2+t_1)}\right] + \frac{d_1}{\theta_2}\left[-\frac{1}{\theta_2} + \frac{1}{\theta_2} e^{\theta_2(T-t_2)} - (T - t_2)\right]\right\} + c_p\{s_r t_1 - dt_2 - d_1(T - t_2)\} + t_1 s_r c_p + O_c$ (7)

Total profit ( $Zp) = s_c\;\eta\{s_r t_1 - dt_2 - d_1(T - t_2)\} + s_p[\int_0^{t_2} d\,dt + \int_{t_2}^{T} d_1 dt] - Total\ cost$

$$= s_c\,\eta\{s_r t_1 - dt_2 - d_1(T-t_2)\} + s_p dt_2 + s_p d_1(T-t_2) - h_c\left\{\frac{s_r-d}{\theta_1}\left[t_1+\frac{e^{-\theta_1 t_1}}{\theta_1}-\frac{1}{\theta_1}\right]-\frac{d}{\theta_2}(t_2-t_1)-\frac{d_1}{\theta_2^2}\left[e^{\theta_2(T-t_2)}-e^{\theta_2(T-t_1)}\right]-\frac{d-d_1}{\theta_2^2}\left[e^{-2t_2\theta_2}-e^{-\theta_2(t_2+t_1)}\right]+\frac{d_1}{\theta_2}\left[-\frac{1}{\theta_2}+\frac{1}{\theta_2}e^{\theta_2(T-t_2)}-(T-t_2)\right]\right\} - 2t_1 s_r c_p + c_p dt_2 + d_1(T-t_2) - O_c \tag{8}$$

Total profit, $z_p = s_c\eta\{s_r t_1 - dt_2 - e^T d(T-t_2)\} + s_p dt_2 + s_p e^T d(T-t_2) - h_c\left[\left\{t_1+\frac{e^{-\theta_1 t_1}}{\theta_1}-\frac{1}{\theta_1}\right\}-\frac{d}{\theta_2}(t_2-t_1)-\frac{e^T d}{\theta_2^2}\left\{e^{\theta_2(T-t_2)}-e^{\theta_2(T-t_1)}\right\}-\frac{d-e^T d}{\theta_2^2}\left\{e^{-2t_2\theta_2}-e^{-\theta_2(t_2+t_1)}\right\}+\frac{e^T d}{\theta_2}\left\{-\frac{1}{\theta_2}+\frac{1}{\theta_2}e^{\theta_2(T-t_2)}-(T-t_2)\right\}\right]\frac{s_r-d}{\theta_1} - 2t_1 s_r c_p + c_p dt_2 + e^T d(T-t_2) - O_c$

$$= d\left[s_c\eta\{-t_2 - e^T(T-t_2)\} + s_p t_2 + s_p e^T(T-t_2) - h_c\left\{-\frac{1}{\theta_1}\left(t_1+\frac{e^{-\theta_1 t_1}}{\theta_1}-\frac{1}{\theta_1}\right)+\frac{t_1-t_2}{\theta_2}-\frac{e^T}{\theta_2^2}\left\{e^{\theta_2(T-t_2)}-e^{-\theta_2(T-t_1)}\right\}-\frac{1-e^T}{\theta_2^2}\left\{e^{-2t_2\theta_2}-e^{-\theta_2(t_2+t_1)}\right\}+\frac{e^T}{\theta_2}\left(-\frac{1}{\theta_2}+\frac{1}{\theta_2}e^{\theta_2(T-t_2)}-(T-t_2)\right\}+c_p t_2+e^T(T-t_2)\right)\right] + \left[s_c s_r\eta t_1 - \frac{h_c s_r}{\theta_1}\left\{t_1+\frac{e^{-\theta_1 t_1}}{\theta_1}-\frac{1}{\theta_1}\right\} - 2t_1 s_r c_p - O_c\right] = d\phi_1 + \phi_2$$

Where $\phi_1 = s_c\eta\{-t_2 - e^T(T-t_2)\} + s_p t_2 + s_p e^T(T-t_2) - h_c\{-\frac{1}{\theta_1}\left(t_1+\frac{e^{-\theta_1 t_1}}{\theta_1}-\frac{1}{\theta_1}\right)+\frac{t_1-t_2}{\theta_2}-\frac{e^T}{\theta_2^2}\left\{e^{\theta_2(T-t_2)}-e^{-\theta_2(T-t_1)}\right\}-\frac{1-e^T}{\theta_2^2}\left\{e^{-2t_2\theta_2}-e^{-\theta_2(t_2+t_1)}\right\}+\frac{e^T}{\theta_2}\left(-\frac{1}{\theta_2}+\frac{1}{\theta_2}e^{\theta_2(T-t_2)}-(T-t_2)\right\}+c_p t_2+e^T(T-t_2)$ (9)

and $\phi_2 = s_c s_r\eta t_1 - \frac{h_c s_r}{\theta_1}\left\{t_1+\frac{e^{-\theta_1 t_1}}{\theta_1}-\frac{1}{\theta_1}\right\} - c_3 - 2t_1 s_r c_p - O_c$ (10)

$$d = \frac{z_p-\varphi_2}{\varphi_1} \tag{11}$$

So, the crisp mathematical problem is

$$\begin{cases} Maximize\ Z_p = d\phi_1 + \phi_2 \\ J_1(t) = \frac{s_r-d}{\theta_1}\left[1-e^{-\theta_1 t}\right] \\ J_2(t) = -\frac{d}{\theta_2} + [\frac{d_1}{\theta_2}e^{\theta_2 T} + \left(\frac{d}{\theta_2}-\frac{d_1}{\theta_2}\right)e^{-\theta_2 t_2}]e^{-\theta_2 t} \\ Subject\ to\ conditions\ (9,10) \end{cases} \tag{12}$$

# 4. Fuzzy Mathematical Model

We know that the demand rate of an inventory problem plays a vital role in maximizing the inventory profit. Generally, the demand rate is uncertain in nature. In this study, we have considered the demand rate as a pentagonal dense fuzzy number $\langle d_0\left(1-\frac{\rho_1}{1+m}\right), d_0\left(1-\frac{\rho_2}{1+m}\right), d_0, d_0\left(1+\frac{\sigma_2}{1+m}\right), d_0\left(1+\frac{\sigma_1}{1+m}\right)\rangle$.

The membership function of the demand rate is defined by

$$\mu(\tilde{d}) = \begin{cases} \dfrac{d-d_0\left(1-\frac{\rho_1}{1+m}\right)}{\frac{d_0(\rho_1-\rho_2)}{1+m}} & for\ d_0\left(1-\frac{\rho_1}{1+m}\right)\le d\le d_0\left(1-\frac{\rho_2}{1+m}\right) \\ \dfrac{d-d_0\left(1-\frac{\rho_2}{1+m}\right)}{\frac{d_0\rho_2}{1+m}} & for\ d_0\left(1-\frac{\rho_2}{1+m}\right)\le d\le d_0 \\ 1 & for\ d=d_0 \\ \dfrac{d_0\left(1+\frac{\sigma_2}{1+m}\right)-d}{\frac{d_0\sigma_2}{1+m}} & for\ d_0\le d\le d_0\left(1+\frac{\sigma_2}{1+m}\right) \\ \dfrac{d_0\left(1+\frac{\sigma_1}{1+m}-d\right)}{\frac{d_0(\sigma_1-\sigma_2)}{1+m}} & for\ d_0\left(1+\frac{\sigma_2}{1+m}\right)\le d\le d_0\left(1+\frac{\sigma_1}{1+m}\right) \\ 0 & otherwise \end{cases}$$

And corresponding index value is given by

$$\mathrm{I}(\tilde{d}) = \frac{\int_{a_1}^{a_2} x\,\mu(x)dx+\int_{a_2}^{a_3} x\,\mu(x)dx+\int_{a_3}^{a_4} x\,\mu(x)dx+\int_{a_4}^{a_5} x\,\mu(x)dx}{\int_{a_1}^{a_2} \mu(x)dx+\int_{a_2}^{a_3} \mu(x)dx+\int_{a_3}^{a_4} \mu(x)dx+\int_{a_4}^{a_5} \mu(x)dx} = \frac{P}{Q}$$

Where $P = \sum_{m=0}^{M}\left\{\int_{d_0(1-\frac{\rho_1}{1+m})}^{d_0(1-\frac{\rho_2}{1+m})} x\left\{\frac{x-d_0(1-\frac{\rho_1}{1+m})}{\frac{d_0(\rho_1-\rho_2)}{1+m}}\right\}dx + \int_{d_0(1-\frac{\rho_2}{1+m})}^{d_0} x\left\{\frac{x-d_0\left(1-\frac{\rho_2}{1+m}\right)}{\frac{d_0\rho_2}{1+m}}\right\}dx + \int_{d_0}^{d_0\left(1+\frac{\sigma_2}{1+m}\right)} x\left\{\frac{d_0\left(1+\frac{\sigma_2}{1+m}\right)-x}{\frac{d_0\sigma_2}{1+m}}\right\}dx + \int_{d_0\left(1+\frac{\sigma_2}{1+m}\right)}^{d_0\left(1+\frac{\sigma_1}{1+m}\right)} x\left\{\frac{d_0\left(1+\frac{\sigma_1}{1+m}\right)-x}{\frac{d_0(\sigma_1-\sigma_2)}{1+m}}\right\}dx\right\}$

$$= \frac{d_0^2(\rho_1-\rho_2)}{\sigma(1+m)}\left[3-\frac{\rho_1+\rho_2}{1+m}\right] + \frac{d_0^2\rho_2}{\sigma(1+m)}\left[3-\frac{\rho_2}{1+m}\right] + \frac{d_0^2\sigma_2}{\sigma(1+m)}\left[3+\frac{\sigma_2}{1+m}\right] + \frac{d_0^2(\sigma_1-\sigma_2)}{\sigma(1+m)}\left[3+\frac{\sigma_1+\sigma_2}{1+m}\right]$$

$$= \frac{d_0^2 3(\rho_1+\sigma_1)}{\sigma(1+m)} - \frac{d_0^2\left(\rho_1^2-\sigma_1^2\right)}{\sigma(1+m)^2}$$

$$Q = \int_{a_1}^{a_2} \mu(x)\,dx + \int_{a_2}^{a_3} \mu(x)\,dx + \int_{a_3}^{a_4} \mu(x)\,dx + \int_{a_4}^{a_5} \mu(x)\,dx$$

$$= \int_{d_0\left(1-\frac{\rho_1}{1+m}\right)}^{d_0\left(1-\frac{\rho_2}{1+m}\right)}\left\{\frac{x-d_0\left(1-\frac{\rho_1}{1+m}\right)}{\frac{d_0(\rho_1-\rho_2)}{1+m}}\right\}dx + \int_{d_0\left(1-\frac{\rho_2}{1+m}\right)}^{d_0}\left\{\frac{x-d_0\left(1-\frac{\rho_2}{1+m}\right)}{\frac{d_0\rho_2}{1+m}}\right\}dx + \int_{d_0}^{d_0\left(1+\frac{\sigma_2}{1+m}\right)}\left\{\frac{d_0\left(1+\frac{\sigma_2}{1+m}\right)-x}{\frac{d_0\sigma_2}{1+m}}\right\}dx + \int_{d_0\left(1+\frac{\sigma_2}{1+m}\right)}^{d_0\left(1+\frac{\sigma_1}{1+m}\right)}\left\{\frac{d_0\left(1+\frac{\sigma_1}{1+m}\right)-x}{\frac{d_0(\sigma_1-\sigma_2)}{1+m}}\right\}dx$$

$$= \sum_{m=0}^{M}\left\{\frac{d_0(\rho_1-\rho_2)}{2(1+m)} + \frac{d_0\rho_2}{2(1+m)} + \frac{d_0\sigma_2}{2(1+m)} + \frac{d_0(\sigma_1-\sigma_2)}{2(1+m)}\right\}$$

$$= \sum_{m=0}^{M}\frac{d_0(\rho_1+\sigma_1)}{2(1+m)}$$

Now, the membership function of $z_p$ is defined by

$$\mu(\widetilde{z_p}) = \begin{cases} \frac{\frac{z_p-\varphi_2}{\varphi_1}-d_0\left(1-\frac{\rho_1}{1+m}\right)}{\frac{d_0(\rho_1-\rho_2)}{1+m}} & for\ \varphi_1 d_0\left(1-\frac{\rho_1}{1+m}\right)+\varphi_2 \le z_p \le \varphi_1 d_0\left(1-\frac{\rho_2}{1+m}\right)+\varphi_2 \\ \frac{\frac{z_p-\varphi_2}{\varphi_1}-d_0\left(1-\frac{\rho_2}{1+m}\right)}{\frac{d_0\rho_2}{1+m}} & for\ \varphi_1 d_0\left(1-\frac{\rho_2}{1+m}\right)+\varphi_2 \le z_p \le \varphi_1 d_0+\varphi_2 \\ 1 & for\ z_p = \varphi_1 d_0+\varphi_2 \\ \frac{d_0\left(1+\frac{\sigma_2}{1+m}\right)-\frac{z_p-\varphi_2}{\varphi_1}}{\frac{d_0\sigma_2}{1+m}} & for\ \varphi_1 d_0+\varphi_2 \le z_p \le \varphi_1 d_o\left(1+\frac{\sigma_2}{1+m}\right)+\varphi_2 \\ \frac{d_0\left(1+\frac{\sigma_1}{1+m}\right)-\frac{z_p-\varphi_2}{\varphi_1}}{\frac{d_0(\sigma_1-\sigma_2)}{1+m}} & for\ \varphi_1 d_0\left(1+\frac{\sigma_2}{1+m}\right) \le z_p \le \varphi_1 d_0\left(1+\frac{\sigma_1}{1+m}\right)+\varphi_2 \\ 0 & ;Otherwise \end{cases}$$

and the corresponding index value is given by

$$\mathrm{I}(\tilde{z}) = \frac{\sum_{m=0}^{M}\left\{\int_{a_1}^{a_2} z\mu(z)dz+\int_{a_2}^{a_3} z\mu(z)dz+\int_{a_3}^{a_4} z\mu(z)dz+\int_{a_4}^{a_5} z\mu(z)dz\right\}}{\sum_{m=0}^{M}\left\{\int_{a_1}^{a_2} \mu(z)dz+\int_{a_2}^{a_3} \mu(z)dz+\int_{a_3}^{a_4} \mu(z)dz+\int_{a_4}^{a_5} \mu(z)dz\right\}} = \frac{P}{Q}$$

where $P = \int_{\varphi_1 d_0\left(1-\frac{\rho_1}{1+m}\right)+\varphi_2}^{\varphi_1 d_0\left(1-\frac{\rho_2}{1+m}\right)+\varphi_2} z\left\{\frac{\frac{z-\varphi_2}{\varphi_1}-d_0\left(1-\frac{\rho_1}{1+m}\right)}{\frac{d_0(\rho_1-\rho_2)}{1+m}}\right\}dz + \int_{\varphi_1 d_0\left(1-\frac{\rho_2}{1+m}\right)+\varphi_2}^{\varphi_1 d_0+\varphi_2} z\left\{\frac{\frac{z-\varphi_2}{\varphi_1}-d_0\left(1-\frac{\rho_2}{1+m}\right)}{\frac{d_0\rho_2}{1+m}}\right\}dz +$

$$\int_{\varphi_1 d_0+\varphi_2}^{\varphi_1 d_0\left(1+\frac{\sigma_2}{1+m}\right)+\varphi_2} z\left\{\frac{d_0\left(1+\frac{\sigma_2}{1+m}\right)-\frac{z-\varphi_2}{\varphi_1}}{\frac{d_0\sigma_2}{1+m}}\right\}dz+\int_{\varphi_1 d_o\left(1+\frac{\sigma_2}{1+m}\right)+\varphi_2}^{\varphi_1 d_0\left(1+\frac{\sigma_1}{1+m}\right)+\varphi_2} z\left\{\frac{d_0\left(1+\frac{\sigma_1}{1+m}\right)-\frac{z-\varphi_2}{\varphi_1}}{\frac{d_0(\sigma_1-\sigma_2)}{1+m}}\right\}dz$$

$$=\frac{1}{6}\left[\frac{\varphi_1 d_0(\rho_1-\rho_2)}{1+m}\left\{3(\varphi_1 d_0+\varphi_2)-\varphi_1 d_0\frac{(\rho_1+2\rho_2)}{1+m}\right\}+\frac{\varphi_1 d_0\rho_2}{1+m}\left\{3(\varphi_1 d_0+\varphi_2)-\frac{\varphi_1 d_0\rho_2}{1+m}\right\}+\frac{\varphi_1 d_0\sigma_2}{1+m}\left\{3(\varphi_1 d_0+\varphi_2)+\frac{\varphi_1 d_0\sigma_2}{1+m}\right\}+\right.$$
$$\left.\frac{\varphi_1 d_0(\sigma_1-\sigma_2)}{1+m}\left\{3(\varphi_1 d_0+\varphi_2)+\varphi_1 d_0\frac{(\sigma_1+2\sigma_2)}{1+m}\right\}\right]$$

$$=\sum_{m=0}^{M}\frac{1}{6}\left[\frac{3(\varphi_1 d_0+\varphi_2)}{1+m}\{\varphi_1 d_0(\rho_1-\rho_2)+\varphi_1 d_0\rho_2+\varphi_1 d_0\sigma_2+\varphi_1 d_0(\sigma_1-\sigma_2)\}-\frac{\varphi_1^2 d_0^2}{(1+m)^2}\{(\rho_1-\rho_2)(\rho_1+2\rho_2)+\rho_2^2-\sigma_2^2-\right.$$
$$\left.(\sigma_1-\sigma_2)(\sigma_1+2\sigma_2)\}\right]$$

$$Q=\sum_{m=0}^{M}\frac{1}{2}\left[\frac{\varphi_1 d_0(\rho_1-\rho_2)}{1+m}+\frac{\varphi_1 d_0\rho_2}{1+m}+\frac{\varphi_1 d_0\sigma_2}{1+m}+\frac{\varphi_1 d_o(\sigma_1-\sigma_2)}{1+m}\right]$$

$$=\sum_{m=0}^{M}\frac{\varphi_1 d_0(\rho_1+\sigma_1)}{2(1+m)}$$

# 5. Numerical Result

Let us consider $h_c=2.5, O_c=500, C_p=500, S_p=700, d=1500, \theta_1=0.004, \theta_2=0.005, S_c=50, \eta=0.3$ then we get the following results.

**Table 1: Optimal solution of our proposed model.**

| Methodology | | $t_1^*$ | $t_2^*$ | $T^*$ | $J_1^*(t_1)$ | $J_2^*(t_2)$ | $Z^*$ |
|---|---|---|---|---|---|---|---|
| Crisp | | 0.242 | 0.666 | 0.75 | 362.73 | 125.03 | 114410.9 |
| PDF | M=1 | 0.232 | 0.625 | 0.74 | 370.23 | 123.21 | 114789.2 |
| | M=2 | 0.251 | 0.645 | 0.76 | 380.45 | 118.14 | 118546.1 |
| | M=3 | 0.249 | 0.656 | 0.75 | 375.21 | 122.36 | 116541.8 |
| | M=4 | 0.240 | 0.638 | 0.74 | 371.25 | 123.89 | 115213.8 |
| Trapezoidal Dense Fuzzy | M=1 | 0.231 | 0.624 | 0.74 | 371.23 | 122.21 | 114889.2 |
| | M=2 | 0.253 | 0.646 | 0.76 | 381.45 | 116.14 | 118246.1 |
| | M=3 | 0.249 | 0.657 | 0.75 | 376.21 | 123.36 | 116441.8 |
| | M=4 | 0.240 | 0.636 | 0.74 | 373.25 | 124.89 | 115413.8 |
| Triangular Dense Fuzzy | M=1 | 0.232 | 0.626 | 0.74 | 371.23 | 123.21 | 114989.2 |
| | M=2 | 0.254 | 0.644 | 0.76 | 381.45 | 117.14 | 118446.1 |
| | M=3 | 0.247 | 0.656 | 0.75 | 376.21 | 123.36 | 116341.8 |
| | M=4 | 0.241 | 0.637 | 0.74 | 372.25 | 124.89 | 115613.8 |

From Table **1**, we see that the Inventory profit gets an optimum value $118546.1 in pentagonal dense fuzzy environment where whereas the Inventory profit becomes $114410.9, $118246.1 and $118446.1 in crisp, trapezoidal dense fuzzy, and triangular dense fuzzy environment. Also, it is clear that in each fuzzy environment, the Inventory profit gives maximum value for M=2.

As we see in Table **1**, the Inventory profit gets maximum value in Pentagonal dense fuzzy environment, for Sensitivity analysis of PDF model, we have changed each parameter from -20% to +20% one by one and get the following result.

**Table 2: Sensitivity analysis of TDF model.**

| Parameters | % Change | $t_1^*$ | $t_2^*$ | $T^*$ | $J_1^*(t_1)$ | $J_2^*(t_2)$ | $Z^*$ |
|---|---|---|---|---|---|---|---|
| $h_c$ | +20 | .2419 | .6667 | .7500 | 362.7277 | 125.0313 | 114388.9 |
| | +10 | .2419 | .6667 | .7500 | 362.7277 | 125.0313 | 114399.9 |
| | -10 | .2419 | .6667 | .7510 | 362.7277 | 125.0313 | 114421.8 |
| | -20 | .2419 | .6667 | .7510 | 362.7277 | 125.0313 | 114432.8 |
| $O_c$ | +20 | .2343 | .6818 | .7500 | 351.3978 | 102.2936 | 122654.5 |
| | +10 | .2419 | .6818 | .7510 | 362.7277 | 102.2936 | 108335.4 |
| | -10 | .2419 | .7317 | .7510 | 362.7277 | 27.4405 | 88598.19 |
| | -20 | .2500 | .7317 | .7510 | 374.8126 | 27.4405 | 81375.7 |
| $C_p$ | +20 | .2419 | .7142 | .7510 | 362.7277 | 53.5771 | 57456.51 |
| | +10 | .2500 | .7143 | .7510 | 374.8126 | 53.5771 | 58718.75 |
| | -10 | .2500 | .7143 | .7510 | 374.8126 | 53.5772 | 101575.9 |
| | -20 | .2500 | .7317 | .7510 | 374.8126 | 27.4405 | 113464.2 |
| $S_p$ | +20 | .2500 | .7317 | .7510 | 374.8126 | 27.4405 | 235111.3 |
| | +10 | .2273 | .7143 | .7510 | 340.7542 | 53.5772 | 206281.7 |
| | -10 | .2273 | .7143 | .7510 | 340.7542 | 53.5772 | 40404.21 |
| | -20 | .2273 | .6818 | .7510 | 340.7542 | 102.2936 | No feasible |
| d | +20 | .2273 | .6818 | .7500 | 272.6033 | 122.7524 | 297960.0 |
| | +10 | .2500 | .7143 | .7500 | 337.3313 | 58.9348 | 162110.5 |
| | -10 | .2344 | .7143 | .7500 | 386.5375 | 48.2195 | 29531.6 |
| | -20 | .2500 | .7317 | .7500 | 449.7751 | 21.9542 | No feasible |
| $\theta_1$ | +20 | .2419 | .6977 | .7500 | 362.6926 | 78.5007 | 102080.0 |
| | +10 | .2344 | .6818 | .7500 | 351.3813 | 102.2936 | 122754.5 |
| | -10 | .2500 | .6818 | .7500 | 374.8313 | 102.2936 | 93058.17 |
| | -20 | .2500 | .7317 | .7500 | 371.8500 | 27.4405 | 73220.33 |
| $\theta_2$ | +20 | .2500 | .7317 | .7500 | 374.8126 | 27.4408 | 161427.9 |
| | +10 | .2500 | .6818 | .7500 | 374.8126 | 102.2957 | 141171.5 |
| | -10 | .2344 | .6818 | .7500 | 351.3978 | 102.2916 | 62111.55 |
| | -20 | .2344 | .6977 | .7500 | 351.3978 | 78.4982 | No feasible |
| $S_c$ | +20 | .2344 | .6977 | .7500 | 351.3978 | 78.5007 | 114920.7 |
| | +10 | .2500 | .6977 | .7500 | 374.8126 | 78.5007 | 86058.63 |
| | -10 | .2273 | .7317 | .7500 | 340.7544 | 27.4405 | 117127.4 |
| | -20 | .2500 | .7317 | .7500 | 374.8126 | 27.4405 | 74427.29 |
| $\eta$ | +20 | .2273 | .7317 | .7500 | 340.7542 | 27.4405 | 114995.1 |
| | +10 | .2273 | .7317 | .7500 | 340.7542 | 27.4405 | 115705.9 |
| | -10 | .2273 | .6977 | .7500 | 340.7542 | 78.5007 | 130743.9 |
| | -20 | .2419 | .6972 | .7500 | 362.7277 | 78.5007 | 103540.6 |

# 6. Graphical Illustrations

Using the data from Table **1**-**2**, we have drawn the following graphs for graphical illustration.

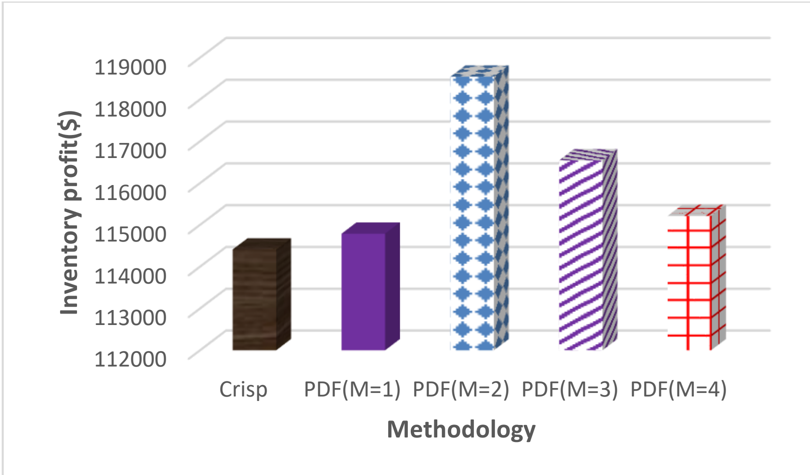


**Figure 3:** Inventory profit vs methodology.

Fig. (**3**) shows the inventory profit under crisp and pentagonal dense fuzzy environment. From this figure, it is clear that the inventory profit becomes maximum in PDF (M=2) environment.

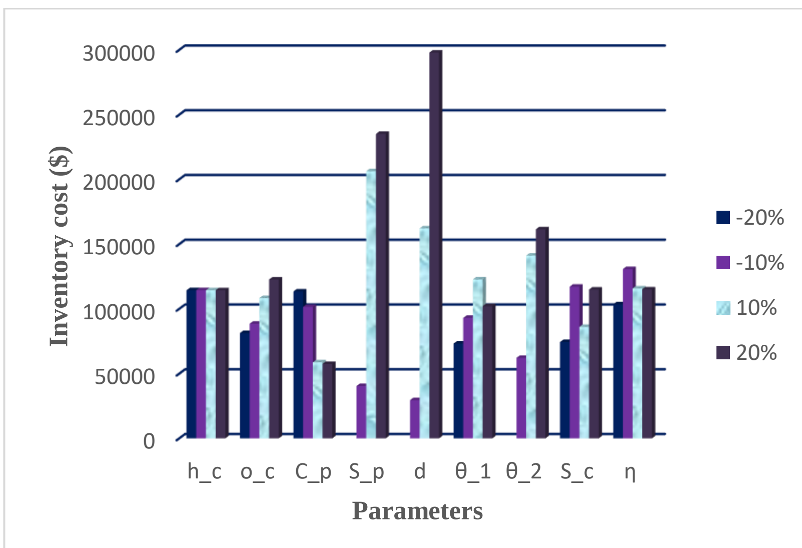


**Figure 4:** Graphical illustration of sensitivity analysis.

From Fig. (**4**), we see that the Selling price $(S_p)$ and demand rate (d) are highly sensitive and $h_c, O_C, S_c$ and $\eta$ are almost insensitive parameters within this variation. Also, deterioration rate $\theta_1$and $\theta_2$ are low-sensitive parameters within this variation.

# 7. Conclusion

In this study, we have discussed an economic order quantity inventory model of deteriorating items under non-random uncertain demand. Here we consider the customers screen the fresh items during the selling period. After a certain period of time, the deteriorated items are sold at a discounted price. Firstly, we solve the crisp model, and then the model is converted into a fuzzy environment. Here we consider the pentagonal dense fuzzy, trapezoidal dense fuzzy and triangular dense fuzzy for a comparative study. Throughout the study, we have seen that the model gets finer optimum in pentagonal dense fuzzy environment. So, the pentagonal dense fuzzy model is much more suitable for decision-makers to make decisions on an inventory problem.

## Conflict of Interest

The authors declare no conflict of interest.

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