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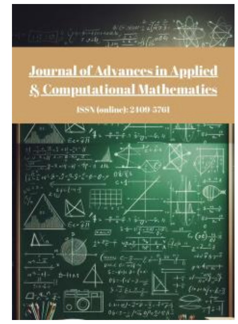

Abstract Dirichlet Problem for Elliptic System on Corner Domain

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ABSTRACT

In this paper, we analyze abstract Dirichlet problem for elliptic system set on singular corner domain. We investigate the existence and uniqueness of strict solutions to the above problem using da Prato-Grisvard theory. The study is performed in the framework of little Hölder spaces.

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1. Introduction and Preliminaries

The study of partial differential equations (PDEs) requires a precise and robust analytical framework, especially when addressing problems in domains with complex geometries or involving low-regularity data [1, 2]. Operator theory has become an indispensable tool in this context, offering a systematic way to formulate and analyze PDEs in abstract functional spaces. By representing differential equations as operator equations in suitable Banach spaces, we can use the well known spectral theory and interpolation techniques to establish existence, uniqueness, and regularity results [3-6].

A particularly effective aspect of this approach is the sum's operators theory. This method is especially valuable when dealing with elliptic or parabolic operators. Classical results, such as those of Kato and Lions, provide criteria under which the sum of sectorial or accretive operators generates an analytic semigroup, ensuring well-posedness even under challenging conditions.

This abstract framework proves especially powerful when studying PDEs in little Hölder spaces, which consist of functions whose derivatives satisfy uniform Hölder continuity with a given exponent [7-9]. Such spaces are natural settings for boundary value problems with Hölder continuous coefficients or data, which frequently arise in physical models featuring material heterogeneities or non-smooth forcing terms. The efficiency of this abstract operator-theoretic approach lies in its generality and adaptability, the operator-sum framework provides a unified methodology that can handle a wide kinds of problems. It allows us to interplay between geometryof the domain , boundary conditions and the regularity of coefficients. In this way, operator theory—particularly the sum of closed linear operators approach combined with the analysis in little Hölder spaces, represents a highly effective strategy for studying PDEs on nonsmooth domains. It bridges the gap between abstract functional analysis and concrete applications, providing powerful tools to rigorously address complex real-world problems.

1.1. Basics of da Prato-Grisvard Theory

Let X be a complex Banach space and let M, N be two closed linear operators on X with domains $Dom(M)$, $Dom(N)$. Let S be the operator defined by

$$\begin{cases} Su := Mu + Nu, \\ u \in Dom(S) := Dom(N) \cap Dom(M), \end{cases} \tag{1.1}$$

where M and N verify the assumptions

$$(H.1)\begin{cases} i)\rho(N) \supseteq \sum_N = \{\mu : |\mu| \geq r, |Arg(\mu)| < \pi - \varepsilon_N\}, \\ \forall \mu \in \sum_N \quad \|(N-\mu I)^{-1}\|_{L(X)} = O\left(\frac{1}{|\mu|}\right). \\ ii)\rho(M) \supseteq \sum_M = \{\mu : |\mu| \geq r, |Arg(\mu)| < \pi - \varepsilon_M\}, \\ \forall \mu \in \sum_M \quad \|(M-\mu I)^{-1}\|_{L(X)} = O\left(\frac{1}{|\mu|}\right). \\ iii)\varepsilon_N + \varepsilon_M < \pi. \\ iv)\overline{Dom(N) + Dom(M)} = X \end{cases}$$

and

$$(H.2)\begin{cases} \forall \mu_1 \in \rho(N), \forall \mu_2 \in \rho(M) \\ (N-\mu_1 I)^{-1}(M-\mu_2 I)^{-1} - (M-\mu_2 I)^{-1}(N-\mu_1 I)^{-1} \\ = [(N-\mu_1 I)^{-1}; (M-\mu_2 I)^{-1}] = 0, \end{cases}$$

where $\rho(N)$ and $\rho(M)$ are the resolvent sets of N and M. Before stating the main result from [3], we recall the following notion:

Definition 1.1 The interpolation spaces $D_N(\rho)$, $\rho \in]0,1[$ are defined as follows

$$D_N(\rho) = \left\{ \xi \in X : \lim_{r \to 0^+} \left\| r^\rho N(N - rI)^{-1} \xi \right\|_X = 0 \right\},$$

For more details about these spaces [10-12]. We have the following:

Theorem 1.2 Let $\rho \in]0,1[$. Assume $(H.1)$, $(H.2)$ and $f \in D_N(\rho)$. Then, the problem

$$Nu + Mu = f,$$

has a unique strict solution

$$u \in Dom(N) \cap Dom(M).$$

given by

$$u = -\frac{1}{2i\pi} \int_\gamma (M + \mu)^{-1} (N - \mu)^{-1} f d\mu,$$

where γ is a sectorial curve lying in $(\Sigma_N) \cap (\Sigma_{-M})$ oriented from $\infty e^{+i\theta_0}$ to $\infty e^{-i\theta_0}$ with $\varepsilon_M < \theta_0 < \pi - \varepsilon_N$.

Moreover, one has

$$Nu, Mu \in D_N(\rho).$$

2. Position of Problem

Le $\Omega_0 \subseteq R^2$ be truncated plane sector defined by

$$\Omega_0 := \{(r \cos\theta, r \sin\theta) \in R^2 : r \in R^+, 0 < \theta < \omega\},$$

where $\omega \neq \pi$ and $\omega < 2\pi$. We assume that

$$\partial\Omega_0 = \Gamma_0' \cup \Gamma_1',$$

where

$$\Gamma_0' = \{(r, 0), r \in R^+\}$$

and

$$\Gamma_1' = \{(r \cos\omega, r \sin\omega) : r \in R^+\}.$$

In Ω_0, we consider the following elliptic system of n equations:

$$\Delta u_i = f_i, \qquad i = 1,2, \dots, n, \tag{2.1}$$

associated with the following boundary conditions

$$u_i|_{\Gamma_1 \cup \Gamma_2} = 0 \qquad i = 1,2, \dots, n, \tag{2.2}$$

where

$$f_i \in h^{2\alpha}(\Omega_0), 0 < 2\alpha < 1, \qquad i = 1,2, \dots, n;$$

here, the little Hölder space $h^{2\alpha}(\Omega_0)$ is defined by

$$h^{2\alpha}(\Omega_0) := \left\{ \varphi \in C^{2\alpha}(\Omega_0) : \lim_{\delta \to 0} \sup_{0 < \|(r-r',\theta-\theta')\| \le \delta} \frac{|\varphi(r,\theta)-\varphi(r',\theta')|}{\|(r-r',\theta-\theta')\|^{2\alpha}} = 0 \right\}.$$

The vector space $h^{2\alpha}(\Omega_0)$ endowed with norm

$$\|\varphi\| := \sup_{(r,\theta)\in\Omega_0} |\varphi(r,\theta)| + \sup_{0 < \|(r-r',\theta-\theta')\| \le \delta} \frac{|\varphi(r,\theta)-\varphi(r',\theta')|}{\|(r-r',\theta-\theta')\|^{2\alpha}}, \quad \varphi \in h^{2\alpha}(\Omega_0),$$

is a complex Banach space.

The application of the system (2.1)-(2.2) is frequently encountered across various fields. For a deeper understanding, the reader is encouraged to consult the following articles, [13] and [14]. At this level, we mention that the solvability of system (2.1)-(2.2) is discussed by using an abstract point of view; cf. also [8, 15-21]. The techniques used here are essentially based on the theory of the sum of linear operators in Banach spaces developed by da Prato and Grisvard in [3]. The choice of such techniques is justified by the fact that this kind of technique ensures the existence and uniqueness of solution for the considered problem as well as the maximum regularity properties of solutions.

First of all, using the polar coordinates we can simply show that the system (2.1) becomes

$$\partial_r^2 u_i(r,\theta) - \frac{1}{r}\partial_r u_i(r,\theta) + \frac{1}{r^2}\partial_\theta^2 u_i(r,\theta) = f_i(r,\theta), \quad i = 1,2,\dots,n. \tag{2.3}$$

Multiplying (2.3) by r^2, we obtain

$$(r\partial_r)^2 u_i(r,\theta) + \partial_\theta^2 u_i(r,\theta) = r^2 f_i(r,\theta), \quad i = 1,2,\dots,n.$$

Using the natural change of variable

$$r = e^t \text{and} r\,\partial_r = \partial_t,$$

the sector Ω_0 is transformed into the infinite strip

$$\Omega = \{(r,\theta): r \in R, 0 < \theta < \omega\}$$

while Γ_0' and Γ_1' are transformed into

$$\Gamma_0 = \{(t,0): t \in R\}$$

and

$$\Gamma_1 = \{(t,\omega): t \in R\}.$$

In new coordinatees, the equation (2.3) is equivalent with

$$(\partial_\theta^2 + \partial_t^2)u_i = e^{2t} f_i, \qquad i = 1,2,\dots,n. \tag{2.4}$$

Let us introduce now the following change of functions

$$\begin{cases} v_i(t,\theta) := e^{-(2+\alpha)t} u_i(e^t,\theta), & i = 1,2,\dots,n, \\ g_i(t,\theta) := e^{-\alpha t} f_i(e^t,\theta), & i = 1,2,\dots,n, \end{cases} \tag{2.5}$$

and let us set $m := 2+\alpha$. Then, (2.4) is written in Ω as follows

$$\partial_\theta^2 v_i(t,\theta) + (\partial_t + m)^2 v_i(t,\theta) = g_i(t,\theta), \quad i = 1,2,\ldots,n; \tag{2.6}$$

we accommpany the equation (2.6) with following boundary conditions:

$$v_i(t,0) = 0, \quad v_i(t,\omega) = 0, \quad i = 1,2,\ldots,n. \tag{2.7}$$

By [15, Proposition 7], we have:

Lemma 2.1 Let $\alpha \in (0,1)$. Then, $f_i \in h^{2\alpha}(\Omega_0) \Leftrightarrow g_i \in h^{2\alpha}(\Omega)$, $i = 1,2,\ldots,n$.

3. Study of Problem (2.6)

In this section, which is divided into two separate subsections, we thoroughly analyze the problem (??).

3.1. The Abstract Formulation of Problem (2.6)

Let

$$E := h_0^{2\alpha}(R) := \{\varphi \in h^{2\alpha}(R) : \varphi(0) = 0\},$$

and let us consider the Banach space

$$X = h^{2\alpha}([0,\omega]; E).$$

The main aim of this section is to clarify the abstract version of problem (2.6).

In the first step, we introduce the folowing vectorial functions:

$$\begin{array}{lll} v_i : [0,\omega] \to E; \theta \to v_i(\theta); & v_i(\theta)(t) = v_i(\theta,t), & i = 1,2,\ldots,n, \\ g_i : [0,\omega] \to E; \theta \to g_i(\theta); & g_i(\theta)(t) = g_i(\theta,t), & i = 1,2,\ldots,n, \end{array}$$

and consider the following closed operators

$$\begin{cases} Dom(D) := \{\varphi \in W^{2,p}(R) \cap C^2(R) : \varphi(0) = 0\}, \\ (D\varphi)(t) := \phi''(t) + 2m\phi'(t) + m^2; \end{cases} \tag{3.1}$$

$$\begin{cases} Dom(A) := \{\psi \in W^{2,p}([0,\omega]; X) \cap C^2([0,\omega]; X) : u(0) = u(\omega) = 0\}, \\ (A\psi)(\theta) := \psi''(\theta), \quad \theta \in 0,\omega]; \end{cases} \tag{3.2}$$

$$\begin{cases} Dom(B) := \{\psi \in X : \psi(\theta) \in Dom(D), \quad \theta \in [0,\omega]\} \\ (B\psi)(\theta) := D(\psi(\theta)), \quad \theta \in [0,\omega]. \end{cases} \tag{3.3}$$

A new version of problem (2.6) is then given by

$$AV(\theta) + BV(\theta) = G(\theta), \tag{3.4}$$

where

$$\begin{cases} A = \begin{bmatrix} A & 0 & \ldots \\ \vdots & \ddots & \\ 0 & & A \end{bmatrix} \\ Dom(A) = \otimes\, Dom(A), \end{cases} \tag{3.5}$$

and

$$\begin{cases} B = \begin{bmatrix} B & 0 & \ldots \\ \vdots & \ddots & \\ 0 & & B \end{bmatrix} \\ Dom(B) = \otimes\, Dom(B), \end{cases} \tag{3.6}$$

while

$$V(\theta)=\begin{bmatrix}v_1(\theta)\\ \vdots\\ v_n(\theta)\end{bmatrix}, G(\theta)=\begin{bmatrix}g_1(\theta)\\ \vdots\\ g_n(\theta)\end{bmatrix}$$

We assume that $V(\theta)$ and $G(\theta)$ are two vector-valued functions defined on the space

$$X := h^{2\alpha}([0,\omega];E)\times h^{2\alpha}([0,\omega];E)\times\ldots\times h^{2\alpha}([0,\omega];E), 0<2\alpha<1,$$

endowed with the following norm

$$\|\mathrm{F}\|_{\mathrm{X}} \coloneqq \max_{1\leq i\leq n}\left(\|g_i\|_{h^{2\alpha}\left([0,\omega];h^{2\alpha}(\mathrm{R}_+)\right)}\right).$$

3.2. On the Spectral Properties of the Operators A and B

First, let us state some useful results concerning the operator A defined by (3.2):

Lemma 3.1 There exists $\varepsilon_A>0$ such that

$$\rho(A)\supseteq\textstyle\sum_A \quad =\{\lambda:|\lambda|\geq r, |Arg(\lambda)|<\pi-\varepsilon_A\}$$

and

$$\|(A-\lambda I)^{-1}\|_{L(X)}=O\left(\frac{1}{|\lambda|}\right), \quad \lambda\in\textstyle\sum_A \quad . \tag{3.7}$$

Proof. The proof is purely technical and, because of that, we will present the highlights, only. First of all, we will solve explicitly the equation

$$\begin{cases}v''(\theta)-\lambda v(\theta)=\varphi(\theta)\\ \quad v(0)=v(\omega)=0.\end{cases} \tag{3.8}$$

For $\lambda\in C\backslash R_-$ the unique solution v of (3.8) is given by

$$v(\theta)=(A-\lambda)^{-1}\varphi(\theta)=\int_0^1 \quad K_{\sqrt{\lambda}}(\theta,s)\phi(s)ds,$$

where

$$K_{\sqrt{\lambda}}(\theta,s):=\begin{cases}\frac{sinh\sqrt{\lambda}(\omega-\theta)\ sinh\sqrt{\lambda}s}{\sqrt{\lambda}\,sinh\,\omega\sqrt{\lambda}}, & \text{if } 0\leq s\leq\theta,\\ \frac{sinh\sqrt{\lambda}(\omega-s)\ sinh\sqrt{\lambda}\theta}{\sqrt{\lambda}\,sinh\,\omega\sqrt{\lambda}}, & \text{if}\theta\leq s\leq\omega.\end{cases}$$

Concerning the estimate (3.7), observe that for $0<\theta^{'}<\theta<\omega$, one has

$$(A-\lambda)^{-1}\varphi(\theta)-(A-\lambda)^{-1}\varphi(\theta')$$

$$=I_1+I_2+I_3,$$

where

$$I_1 \quad :=\int_0^{\theta'} \quad \left(\frac{\left(sinh\sqrt{\lambda}(\omega-\theta)-sinh\sqrt{\lambda}(\omega-\theta')\right)\ sinh\sqrt{\lambda}s}{\sqrt{\lambda}\,sinh\,\omega\sqrt{\lambda}}\right)\phi(s)ds,$$

$$I_2 \quad :=\int_{\theta'}^{\theta} \quad \left(\frac{sinh\sqrt{\lambda}(\omega-\theta)\ sinh\sqrt{\lambda}s-sinh\sqrt{\lambda}(\omega-s)\ sinh\sqrt{\lambda}\theta}{\sqrt{\lambda}\,sinh\,\omega\sqrt{\lambda}}\right)\phi(s)ds,$$

$$I_3 \quad :=\int_{\theta}^{1} \quad \frac{sinh\sqrt{\lambda}(\omega-s)\ \left(sinh\sqrt{\lambda}\theta-sinh\sqrt{\lambda}\theta'\right)}{\sqrt{\lambda}\,sinh\,\omega\sqrt{\lambda}}\phi(s)ds.$$

By adapting the same techniques as in [22] and [23], we obtain that

$$\|I_j\|_{h^{2\alpha}([0,\omega]);X)} = O\left(\frac{1}{|\lambda|}\right), \qquad j = 1,2,3,$$

which implies that

$$\|(A-\lambda I)^{-1}\|_{h^{2\alpha}([0,\omega]);X)} = O\left(\frac{1}{|\lambda|}\right).$$

On the other hand, by using a classical argument of analytic continuation of the resolvent, we deduce that there exists $\varepsilon_A \in]0,\pi[$ such that the previous estimate holds true in the sector

$$\sum\nolimits_A = \{\lambda: |\lambda| \geq r, |Arg(\lambda)| < \pi - \varepsilon_A\}.$$

This completes the proof of result.

Concerning the operator B, it is necessary to emphasize that it has the same properties as its realization D:

Lemma 3.2 There exists $\varepsilon_D > 0$ such that

$$\rho(B) \supseteq \sum\nolimits_B = \{\lambda: |\lambda| \geq r, |Arg(\lambda)| < \pi - \varepsilon_B\}$$

and

$$\|(B-\lambda I)^{-1}\|_{L(X)} = O\left(\frac{1}{|\lambda|}\right), \quad \lambda \in \sum\nolimits_B \quad . \tag{3.9}$$

Proof. We have $D = P(C)$, where P is the polynomial

$$P(z) = z^2 + 2mz + m^2 = (z+m)^2.$$

Using the spectral mapping theorem, we easely deduce that there exists $\varepsilon_B \in]0,\pi/2[$ such that

$$\rho(D) \supseteq \sum\nolimits_B = \{\lambda: |\lambda| \geq m^2, |Arg(\lambda)| < \pi - \varepsilon_B\}.$$

On the other hand, for all given complex numbers in this sector, the equation

$$P(z) = \lambda$$

has exactly two complex roots

$$z_\pm = -m \pm \sqrt{\lambda},$$

which implies that

$$(D-\lambda I)^{-1} = (C - z_+)^{-1}(C - z_-)^{-1}.$$

Furthermore, we know that $\sigma(C) = iR$ and

$$[(C+\mu I)^{-1}\phi](t,s) = \begin{cases} -\int_t^{+\infty} e^{\mu(s-t)}\phi(s,\sigma)ds \text{if} \Re\mu < 0, \\ \int_{-\infty}^t e^{\mu(t-s)}\phi(s,\sigma)ds \text{if} \Re\mu > 0, \end{cases}$$

from which we easily obtain the estimate

$$\forall \mu \notin iR: \|(C+\mu I)^{-1}\|_{L(h^{2\alpha}(R_+))} = O\left(\frac{1}{|\Re\mu|}\right),$$

which implies that

$$\|(D-\lambda I)^{-1}\|_{L(h^{2\alpha}(R))}=O\left(\frac{1}{(\Re\mu)^2}\right).$$

Remark 3.3 Here, it is important to note the following facts:

1. A and B are densely defined on X, for more details see Corollary 2.2.3 in [24].

2. A and B commute. More precisely, for all $\mu_1 \in \rho(A), \forall \mu_2 \in \rho(B)$, one has

$$(A-\mu_1 I)^{-1}(B-\mu_2 I)^{-1}-(B-\mu_2 I)^{-1}(A-\mu_1 I)^{-1}.$$

3. From Theorem 3.1.12 in [24], we know that

$$D_A(\alpha)=h^{2\alpha}([0,\omega];E), 0<2\alpha<1.$$

Exploiting all previous results, we are able to state the following spectral properties of the matrix differential operators A and B:

Lemma 3.4 The closed linear operators A and B defined respectively by (3.5) and (3.6) satisfy the following properties:

1. $\exists \varepsilon_A>0: \rho(A)\supseteq \sum_A \quad =\{\lambda: |\lambda|\geq r, |Arg(\lambda)|<\pi-\varepsilon_A\}$ and

$$\|(A-\lambda I)^{-1}\|_{L(X)}=O\left(\frac{1}{|\lambda|}\right), \quad \lambda\in\sum_A \quad .$$

2. $\exists \varepsilon_B>0: \rho(A)\supseteq \sum_A \quad =\{\lambda: |\lambda|\geq r, |Arg(\lambda)|<\pi-\varepsilon_B\}$

$$\|(B-\lambda I)^{-1}\|_{L(X)}=O\left(\frac{1}{|\lambda|}\right), \quad \lambda\in\sum_B \quad .$$

Keeping in mind suitable values of angles ε_A and ε_B, Theorem 1.2 allows us to state our main result concerning the matrix differential equation (3.4):

Theorem 3.5 Let A and B be two operators defined respectively by by (3.5) and (3.6). Then, the problem (3.4) has a unique strict solution

$$V\in Dom(A)\cap Dom(B),$$

which is given by

$$V:=-\frac{1}{2i\pi}\int_\gamma \quad (A+\mu)^{-1}(B-\mu)^{-1}G d\mu,$$

where γ is a sectorial curve lying in $(\sum_A \quad)\cap(\sum_{-B} \quad)$ oriented from $\infty e^{+i\theta_0}$ to $\infty e^{-i\theta_0}$ with $\varepsilon_M<\theta_0<\pi-\varepsilon_N$.

Moreover, we have

$$AV,\ BV\in D_A(\alpha).$$

For more details and also concret examples about these interplation spaces, we refer the reader to [3, 11, 24]. We refrer the reader also to [25-29]. Applying these results to our concrete problem (2.6)-(2.7), we obtain:

Theorem 3.6 Let $\left(g_1(\theta,t),\ldots,g_n(\theta,t)\right)\in\otimes\, h^\alpha([0,\omega];h_0^\alpha(R))$ with $\alpha\in(0,1)$. Then, under conditions (2.7), Problem (2.6) has a unique strict solution

$$\left(v_1(\theta,t),\ldots,v_n(\theta,t)\right)\in\otimes\, C^2([0,\omega];h_0^\alpha(R))$$

such that

$$\left(\partial_\theta^2 v_1(\theta,t),\ldots,\partial_\theta^2 v_n(\theta,t)\right) \in\otimes\, C^2([0,\omega];h_0^\alpha(R))$$

and

$$\left(\partial_t^2 v_1(\theta,t),\ldots,\partial_t^2 v_n(\theta,t)\right) \in\otimes\, C^2([0,\omega];h_0^\alpha(R)).$$

Finally, we state the main result of this paper:

Theorem 3.7 Let $(f_1,\ldots,f_n) \in\otimes\, h^{2\alpha}([0,\omega];h_0^{2\alpha}(R_+))$, $0<2\alpha<1$. Then, under conditions (2.2), Problem (2.1) has a unique strict solution

$$(u_1,\ldots,u_n) \in\otimes\, C^2([0,\omega];h_0^{2\alpha}(R_+)),$$

such that

$$(\partial_\theta^2 u_1,\ldots.,\partial_\theta^2 u_n) \in\otimes\, C^2([0,\omega];h_0^{2\alpha}(R_+))$$

and

$$(\partial_r^2 u_1,..,\partial_r^2 u_n) \in\otimes\, C^2([0,\omega];h_0^{2\alpha}(R_+)).$$

Conflict of Interest

The authors declare there is no conflict of interest.

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