

Selected Phenomena of Spontaneous Electric Noise

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Abstract: Recently a modified Drude model of the valence electron gas in metals was investigated mathematically. The implications of this model suggest that the thermal noise voltage increases for example with the length of the metallic conductor. This observation prompted us to carry out some innovative experiments whose outcomes confirmed qualitatively the Drude model. We discuss some implications of this model in the light of the performed measurements.

Keywords: Stochastic process, Kirchhoff's law, noise voltage, noise current.

1. INTRODUCTION

In [1-7] we are concerned with electric noise. In our approach we rely on mathematical models, on statistical inference, which is applied to data obtained from simulation experiments, and on empirical measurements.

Recently, cf. [7], we confirmed empirically that the noise voltage can be in some sense rectified and cumulated, which raises the question whether it is technically possible to cumulate the spontaneous noise current as well.

In the present contribution we consider circuits consisting of solenoids. We connect the solenoids in series and in parallel and measure the resulting noise AC voltages and the noise short currents. For the noise voltages there is an approximate formula, which we discuss in the light of our empirical results.

We believe that the considerations could open an application potential in the field of Energy Harvesting.

2. THE NOISE VOLTAGE IN COPPER SOLENOIDS

In [1] a Drude model of the valence electron gas in metals is considered where the Coulomb interaction between the electrons is neglected. In [1] we derived the formula,

$$\mathbb{V}(U(t)) = \frac{e^2 s^2}{\tau^2 a^4} \cdot 4\lambda\tau \int_0^L (L-h) \cdot p(h) dh \quad (1)$$

for the variance of the voltage, where the parameter $p(h)$ of the thinning can be expressed in terms of the cumulative distribution function Φ of the standard normal distribution *via* the formula,

$$p(h) = \exp\left(-\frac{h^2}{2\sigma^2\tau^2}\right) - \sqrt{2\pi} \cdot \frac{h}{\sigma\tau} \cdot \left(1 - \Phi\left(\frac{h}{\sigma\tau}\right)\right). \quad (2)$$

For big L taking the leading terms of asymptotics, we obtain the approximate formula,

$$\mathbb{V}(U(t)) \approx \varrho e^2 s^2 \sigma^2 \cdot \frac{L}{a^2} \quad (t \geq 0). \quad (2.1)$$

(2.1) is valid if we assume that the thermal voltage signal $(U(t))_{t \geq 0}$ between the ends of a wire is modeled as a trajectory of a stationary stochastic process. The quantities appearing in (2.1) (and above) are explained in Table 1.

Table 1: Legend to (2.1)

ϱ	Material specific density of the electron gas
e	Modulus of the charge of an electron in C
s	Resistivity of the metal in $\Omega \cdot m$
σ^2	Temperature dependent Maxwellian variance of the electron velocity
L	Length of the wire in m
a^2	Cross-section area of the wire in m^2

These quantities are generally accessible for the most of metals, cf. [6] for details.

As an indicator of the strength of the voltage signal we propose the dispersion,

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$$\bar{U} := \left(\mathbb{V}(U(t)) \right)^{\frac{1}{2}} \approx e \cdot s \cdot \sigma \cdot \left(\frac{\rho \cdot L}{a^2} \right)^{\frac{1}{2}} \quad (2.2)$$

which can also be interpreted as the energetic value of the AC voltage $(U(t))_{t \geq 0}$.

(2.2) suggests that \bar{U} depends on the material and on the geometric form of the conductor for a fixed temperature $T = 300 K$. Let us standardize the form of the conductor according to:

$$L = 1500 \text{ m}, \quad a^2 = \pi \cdot r^2 \quad \text{and} \quad r = 2.5 \cdot 10^{-5} \text{ m.} \quad (2.3)$$

For a copper wire whose geometric form is specified in (2.3) we obtain from (2.2) the theoretic value:

$$\bar{U} = 1.8442 \cdot 10^{-2} \text{ V} \quad (2.4)$$

of the thermal voltage.

The measuring of AC voltages is usually restricted to a frequency band, which is only a fraction of the whole noise spectrum. For our measuring device the producer states the band $[18\text{Hz}, 10^6\text{Hz}]$. We consider a copper wire which is wound on a solenoid and whose length and diameter are specified in (2.3). An exemplary AC voltage measurement yields:

$$\bar{U}_{\text{emp}} = 6.3 \cdot 10^{-4} \text{ V}, \quad (2.5)$$

which is smaller than the theoretical value in (2.3) by the factor of 30, which suggests that the actual spectrum of the noise signal is much wider than the frequency band measured out by our device.

3. THE NOISE VOLTAGE IN SOLENOID CIRCUITS

We connect N identical copper wires in a series. For the description of the resulting noise signal we consider N stochastically independent copies:

$$\left(U_t^{(1)} \right)_{t \geq 0}, \dots, \left(U_t^{(N)} \right)_{t \geq 0}$$

of a centered and stationary stochastic process $\left(U_t \right)_{t \geq 0}$ whose variance $\overline{U^2}$ is finite. According to Kirchhoff's law the sum:

$$S_t := U_t^{(1)} + \dots + U_t^{(N)} \quad (t \geq 0) \quad (3)$$

corresponds to the voltage signal between the ends of the series of the solenoids. Clearly, the squared dispersion $\overline{S_N^2}$ of S_t is given by:

$$\overline{S_N^2} = \text{Var}(S_t) = N \cdot \overline{U^2}. \quad (3.1)$$

This means that the resulting voltage is proportional to the square root of N .

For the empirical check of (3.1) we vary the number N of solenoids connected in series and measure the AC voltage. Our measurements are protocolled in the second row of Table 2, which approximatively confirms the validity of (3.1).

In another experiment we vary the number of solenoids connected in parallel and protocol the voltages in the third row of Table 2.

Table 2: Exemplary Empirical AC Voltages

N	1	4	16	Connection
$\bar{U}_{\text{emp}} / \mu\text{V}$	570	930	1500	In series
$\bar{U}_{\text{emp}} / \mu\text{V}$	700	240	90	Parallel

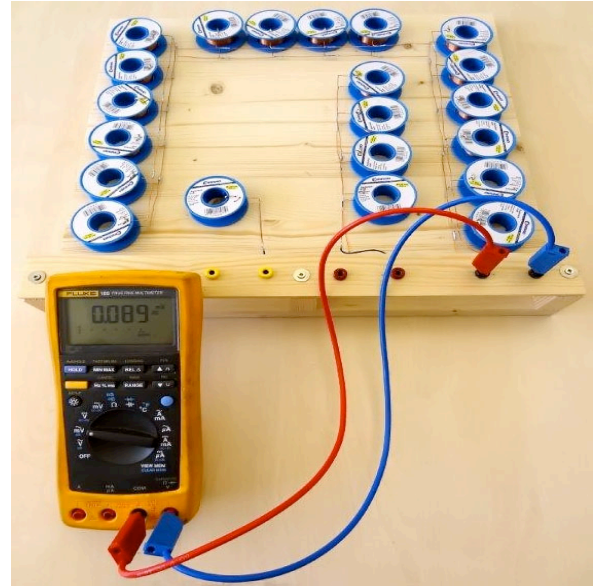


Figure 1: Solenoids connected in parallel.

Although the particular values in Table 2 are exemplary, the following two phenomena can be reproduced:

1. When the solenoids are connected in series, the AC voltage increases with N .
2. When the solenoids are connected in parallel, the AC voltage decreases with $\bar{U}_{\text{emp}} / \mu\text{V}$.

These phenomena comply qualitatively with formula (2.2).

4. SOME SHORT CURRENTS

The short current of a particular source is comparatively easy to be measured by connecting the source to an ampere-meter. The producer of our device states the frequency band [18Hz, 10⁶Hz] for measuring the AC. Like in Section 3 we have varied the number N of solenoids and connected them in series and in parallel and protocolled the results in Table 3.

Table 3: Empirical Alternating Short Currents

N	1	4	16	Connection
$I_{\text{emp}} / \text{nA}$	335	335	335	In series
$I_{\text{emp}} / \text{nA}$	330	330	320	Parallel

The entries in Table 3 suggest that it is difficult to raise the noise current as opposed to the noise voltage. According to our knowledge there is no a theoretical possibility of predicting the measured AC values although they are approximatively reproducible.

CONCLUSION

Between the ends of a long thin isolated wire an AC voltage is generated. This voltage is qualitatively predicted by a formula, which is deduced from a Drude model of a metal. This formula is qualitatively confirmed by connecting solenoids in series and in parallel and measuring the AC voltage in the frequency band [18Hz, 10⁶Hz]. It turns out that the noise voltage can be raised by increasing the length or/and reducing the cross-section area of the wire as opposed to the current, which stabilizes itself at the value 330nA and is not influenced by the number of connected solenoids.

The reported measurements indicate that solenoids can accumulate small net charges, which seem to have an effect on the noise voltage but not on the noise current.

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