On Special Fuzzy Differential Subordinations Using Generalized Sălăgean Operator and Ruscheweyh Derivative

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Abstract: In the present paper we establish several fuzzy differential subordinations regardind the operator $RD_{\lambda,\alpha}^m$, given by $RD_{\lambda,\alpha}^m : A \to A$, $RD_{\lambda,\alpha}^m f(z) = (1-\alpha)R^m f(z) + \alpha D_\lambda^m f(z)$, where $R^m f(z)$ denote the Ruscheweyh derivative, $D_\lambda^m f(z)$ is the generalized Sălăgean operator and $A = \{f \in H(U), f(z) = z + a_2 z^2 + \ldots, z \in U\}$ is the class of normalized analytic functions. A certain fuzzy class, denoted by $RD_m^F(\delta,\lambda,\alpha)$, of analytic functions in the open unit disc is introduced by means of this operator. By making use of the concept of fuzzy differential subordination we will derive various properties and characteristics of the class $RD_m^F(\delta,\lambda,\alpha)$. Also, several fuzzy differential subordinations are established regarding the operator $RD_{\lambda,\alpha}^m$.

Keywords: Fuzzy differential subordination, convex function, fuzzy best dominant, differential operator, generalized Sălăgean operator, Ruscheweyh derivative.

1. INTRODUCTION

One of the most recently study methods in the one complex variable functions theory is the admissible functions method known as "the differential subordination method" introduced by S.S. Miller and P.T. Mocanu in [12], [13] and developed in [14]. The application of this method allows to one obtain some special results and to prove easily some classical results from this domain. More results obtained by the differential subordinations method are differential inequalities. From the development of this method has been written a large number of papers and monographs in the one complex variable functions theory domain.

A justification of the introduction of the differential subordinations theory was presented in [15], "knowing the properties of differential expression for a function we can determine the properties of that function on a given interval." By publication of the papers [15] and [16] the authors wanted to launch a new research direction in mathematics that combines the notions from the complex functions domain with the fuzzy sets theory.

In the same way as mentioned, the author can justify that by knowing the properties of a differential expression on a fuzzy set for a function one can be determined the properties of that function on a given fuzzy set. The author has analyzed the case of one complex functions, leaving as "open problem" the case of real functions.

The author is aware that this new research alternative can be realized only through the joint effort of researchers from both domains. The "open problem" statement leaves open the interpretation of some notions from the fuzzy sets theory such that each one interpret them personally according to their scientific concerns, making this theory more attractive.

The notion of fuzzy subordination was introduced in [15]. In [16] the authors have defined the notion of fuzzy differential subordination. In this paper we will study fuzzy differential subordinations obtained with the differential operator defined in [4].

Denote by U the unit disc of the complex plane, $U = \{z \in C : |z| < 1\}$ and $H(U)$ the space of holomorphic functions in U.

Let $A = \{f \in H(U) : f(z) = z + a_2 z^2 + \ldots, z \in U\}$ and $H[a,n] = \{f \in H(U) : f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \ldots, z \in U\}$ for $a \in C$.

Denote by $K = \left\{ f \in A : \operatorname{Re} \frac{z f''(z)}{f'(z)} + 1 > 0, z \in U \right\}$, the class of normalized convex functions in U.

In order to use the concept of fuzzy differential subordination, we remember the following definitions:

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Definition 1.1 [11] A pair (A,F_A), where $F_A : X \to 0,1]$ and $A = \{x \in X : 0 < F_A(x) \le 1\}$ is called fuzzy subset of X. The set A is called the support of the fuzzy set (A,F_A) and F_A is called the membership function of the fuzzy set (A,F_A). One can also denote $A = \mathrm{supp}(A,F_A)$.

Remark 1.1 [8] In the development work we use the following notations for fuzzy sets:

$$F_{f(D)}\left(f(z)\right) = \text{supp}$$
$$\left(f(D), F_{f(D)}\cdot\right) = \{z \in D : 0 < F_{f(D)}f(z) \le 1\},$$

$$F_{p(U)}p(z) = \text{supp}$$
$$\left(p(U), F_{p(U)}\cdot\right) = \{z \in U : 0 < F_{p(U)}\left(p(z)\right) \le 1\}.$$

We give a new definition of membership function on complex numbers set using the module notion of a complex number $z = x + iy,\ x,y \in R$, $|z| = \sqrt{x^2 + y^2} \ge 0$.

Example 1.1 *Let* $F : C \to R_+$ *a function such that* $F_C(z) = |F(z)|$, $\forall$ $z \in C$. *Denote by* $F_C(C) = \{z \in C : 0 < F(z) \le 1\} = \{z \in C : 0 < |F(z)| \le 1\} =$ *sup* $p(C, F_C)$ *the fuzzy subset of the complex numbers set. We call the subset* $F_C(C) = U_F(0,1)$ *the fuzzy unit disk.*

Definition 1.2 ([15]) Let $D \subset C$, $z_0 \in D$ be a fixed point and let the functions $f, g \in H(D)$. The function f is said to be fuzzy subordinate to g and write $f \prec_F g$ or $f(z) \prec_F g(z)$, if are satisfied the conditions:

1. $f(z_0) = g(z_0)$,

2. $F_{f(D)}f(z) \le F_{g(D)}g(z)$, $z \in D$.

Definition 1.3 ([16, Definition 2.2]) Let $\psi : C^3 \times U \to C$ and h univalent in U, with $\psi(a,0;0) = h(0) = a$. If p is analytic in U, with $p(0) = a$ and satisfies the (second-order) fuzzy differential subordination

$$F_{\psi\left(C^3 \times U\right)}\psi(p(z), zp'(z), z^2p''(z); z) \le F_{h(U)}h(z), \quad z \in U, \tag{1.1}$$

then p is called a fuzzy solution of the fuzzy differential subordination. The univalent function q is called a fuzzy dominant of the fuzzy solutions of the fuzzy differential subordination, or more simple a fuzzy dominant, if $F_{p(U)}p(z) \le F_{q(U)}q(z)$, $z \in U$, for all p satisfying (1.1). A fuzzy dominant $\tilde{q}$ that satisfies $F_{\tilde{q}(U)}\tilde{q}(z) \le F_{q(U)}q(z)$, $z \in U$, for all fuzzy dominants q of (1.1) is said to be the fuzzy best dominant of (1.1).

Lemma 1.1 *([14, Corollary 2.6g.2, p. 66]) Let* $h \in A$ *and* $L[f](z) = G(z) = \frac{1}{z}\int_0^z h(t)\,dt$, $z \in U$. *If* $Re\left(\frac{zh''(z)}{h'(z)} + 1\right) > -\frac{1}{2}$, $z \in U$, *then* $L(f) = G \in K$.

Lemma 1.2 ([17]) Let h be a convex function with $h(0) = a$, and let $\gamma \in C^*$ be a complex number with $Re\,\gamma \ge 0$. If $p \in H[a,n]$ with $p(0) = a$, $\psi : C^2 \times U \to C$, $\psi\left(p(z), zp'(z); z\right) = p(z) + \frac{1}{\gamma}zp'(z)$ an analytic function in U and;

$$F_{\psi\left(C^2 \times U\right)}\left(p(z) + \frac{1}{\gamma}zp'(z)\right) \le F_{h(U)}h(z), \text{i.e.}\, p(z) + \frac{1}{\gamma}zp'(z) \prec_F h(z), \quad z \in U, \tag{1.2}$$

then $F_{p(U)}p(z) \le F_{g(U)}g(z) \le F_{h(U)}h(z)$, i.e. $p(z) \prec_F g(z) \prec_F h(z)$, $z \in U$, where $g(z) = \frac{\gamma}{nz^{\gamma/n}}\int_0^z h(t)t^{\gamma/n-1}dt$, $z \in U$. The function q is convex and is the fuzzy best dominant.

Lemma 1.3 ([17]) Let g be a convex function in U and let $h(z) = g(z) + n\alpha zg'(z)$, $z \in U$, where $\alpha > 0$ and n is a positive integer.

If $p(z) = g(0) + p_nz^n + p_{n+1}z^{n+1} + \ldots,\ z \in U$, is holomorphic in U and $F_{p(U)}\left(p(z) + \alpha zp'(z)\right) \le F_{h(U)}h(z)$, i.e. $p(z) + \alpha zp'(z) \prec_F h(z)$, $z \in U$, then $F_{p(U)}p(z) \le F_{g(U)}g(z)$, i.e. $p(z) \prec_F g(z)$, $z \in U$, and this result is sharp.

We use the following differential operators.

Definition 1.4 *(Al Oboudi [10]) For* $f \in A$, $\lambda \ge 0$ *and* $m \in N$, *the operator* D_λ^m *is defined by* $D_\lambda^m : A \to A$,

$$D_\lambda^0 f(z) = f(z)$$

$$D_\lambda^1 f(z) = (1-\lambda)f(z) + \lambda zf'(z) = D_\lambda f(z), \ldots$$

$$D_\lambda^{m+1} f(z) = (1-\lambda)D_\lambda^m f(z) + \lambda z\left(D_\lambda^m f(z)\right)' = D_\lambda\left(D_\lambda^m f(z)\right), \in U.$$

Remark 1.2 *If* $f \in A$ *and* $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$, *then* $D_\lambda^m f(z) = z + \sum_{j=2}^{\infty} \left[1 + (j-1)\lambda\right]^m a_j z^j$, $z \in U$.

Remark 1.3 For $\lambda = 1$ in the above definition we obtain the Sălăgean differential operator [19].

Definition 1.5 *(Ruscheweyh [18]) For* $f \in A$, $m \in N$, *the operator* R^m *is defined by* $R^m : A \to A$,

$$R^0 f(z) = f(z)$$

$$R^1 f(z) = zf'(z), \ldots$$

$$(m+1) R^{m+1} f(z) = z\left(R^m f(z)\right)' + mR^m f(z), \quad z \in U.$$

Remark 1.4 *If* $f \in A$, $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$, *then* $R^m f(z) = z + \sum_{j=2}^{\infty} C_{m+j-1}^m a_j z^j$, $z \in U$.

Definition 1.6 ([4]) Let $\alpha, \lambda \geq 0$, $m \in N$. Denote by $RD_{\lambda,\alpha}^m$ the operator given by $RD_{\lambda,\alpha}^m : A \to A$, $RD_{\lambda,\alpha}^m f(z) = (1-\alpha) R^m f(z) + \alpha D_\lambda^m f(z), \quad z \in U.$

Remark 1.5 *If* $f \in A$, $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$, *then*

$$RD_{\lambda,\alpha}^m f(z) = z + \sum_{j=2}^{\infty} \left\{ \alpha \left[1 + (j-1)\lambda\right]^m + (1-\alpha) C_{m+j-1}^m \right\} a_j z^j,$$

$z \in U$.

Remark 1.6 For $\alpha = 0$, $RD_{\lambda,0}^m f(z) = R^m f(z)$, $z \in U$, and for $\alpha = 1$, $RD_{\lambda,1}^m f(z) = D_\lambda^m f(z)$, $z \in U$. For $\lambda = 1$, we obtain $RD_{1,\alpha}^m f(z) = L_\alpha^m f(z)$ which was studied in [1], [2], [5]. For $m = 0$, $RD_{\lambda,\alpha}^0 f(z) = (1-\alpha) R^0 f(z) + \alpha D_\lambda^0 f(z) = f(z) = R^0 f(z) = D_\lambda^0 f(z)$, $z \in U$. The operator $RD_{\lambda,\alpha}^m$ was studied in [3], [4], [6], [7].

Remark 1.7 This paper is a particular case for [9].

2. MAIN RESULTS

Using the operator $RD_{\lambda,\alpha}^m$ defined in Definition 1.6 we define the class $RD_m^F(\delta, \lambda, \alpha)$ and we study fuzzy subordinations.

Definition 2.1 [8] Let $f(D) = \text{supp}\left(f(D), F_{f(D)}\right) = \{z \in D : 0 < F_{f(D)} f(z) \leq 1\}$, where $F_{f(D)} \cdot$ is the membership function of the fuzzy set $f(D)$ asociated to the function f. The membership function of the fuzzy set $(\mu f)(D)$ associated to the function μf coincide with the membership function of the fuzzy set $f(D)$ asociated to the function f, i.e. $F_{(\mu f)(D)}\left((\mu f)(z)\right) = F_{f(D)} f(z)$, $z \in D$. The membership function of the fuzzy set $(f+g)(D)$ associated to the function $f+g$ coincide with the half of the sum of the membership functions of the fuzzy sets $f(D)$, respectively $g(D)$, asociated to the function f, respectively g, i.e. $F_{(f+g)(D)}\left((f+g)(z)\right) = \dfrac{F_{f(D)} f(z) + F_{g(D)} g(z)}{2}$, $z \in D$.

Remark 2.1 [8] $F_{(f+g)(D)}\left((f+g)(z)\right)$ can be defined in other ways. Since $0 < F_{f(D)} f(z) \leq 1$ and $0 < F_{g(D)} g(z) \leq 1$, it is evidently that $0 < F_{(f+g)(D)}\left((f+g)(z)\right) \leq 1$, $z \in D$.

Definition 2.2 Let $\delta \in [0,1)$, $\alpha, \lambda \geq 0$ and $m \in N$. A function $f \in A$ is said to be in the class $RD_m^F(\delta, \lambda, \alpha)$ if it satisfies the inequality;

$$F_{\left(RD_{\lambda,\alpha}^m f\right)'(U)} \left(RD_{\lambda,\alpha}^m f(z)\right)' > \delta, \quad z \in U. \qquad (2.1)$$

Theorem 2.1 The set $RD_m^F(\delta, \lambda, \alpha)$ is convex.

Proof. Let the functions $f_j(z) = z + \sum_{j=2}^{\infty} a_{jk} z^j$, $k = 1, 2$, $z \in U$, be in the class $RD_m^F(\delta, \lambda, \alpha)$. It is sufficient to show that the function $h(z) = \eta_1 f_1(z) + \eta_2 f_2(z)$ is in the class $RD_m^F(\delta, \lambda, \alpha)$, with η_1 and η_2 nonnegative such that $\eta_1 + \eta_2 = 1$.

We have $h'(z) = (\mu_1 f_1 + \mu_2 f_2)'(z) = \mu_1 f_1'(z) + \mu_2 f_2'(z)$, $z \in U$, and

$$\left(RD_{\lambda,\alpha}^m h(z)\right)' = \left(RD_{\lambda,\alpha}^m \left(\mu_1 f_1 + \mu_2 f_2\right)(z)\right)'$$
$$= \mu_1 \left(RD_{\lambda,\alpha}^m f_1(z)\right)' + \mu_2 \left(RD_{\lambda,\alpha}^m f_2(z)\right)'.$$

From Definition 2.1 we obtain that

$$F_{\left(RD_{\lambda,\alpha}^m h\right)'(U)} \left(RD_{\lambda,\alpha}^m h(z)\right)' =$$

$$F_{\left(RD_{\lambda,\alpha}^m (\mu_1 f_1 + \mu_2 f_2)\right)'(U)} \left(RD_{\lambda,\alpha}^m \left(\mu_1 f_1 + \mu_2 f_2\right)(z)\right)' =$$

$$F_{\left(RD_{\lambda,\alpha}^m (\mu_1 f_1 + \mu_2 f_2)\right)'(U)} \left(\mu_1 \left(RD_{\lambda,\alpha}^m f_1(z)\right)' + \mu_2 \left(RD_{\lambda,\alpha}^m f_2(z)\right)'\right) =$$

$$\frac{F_{\left(\mu_1 RD^m_{\lambda,\alpha}f_1\right)'(U)}\left(\mu_1\left(RD^m_{\lambda,\alpha}f_1(z)\right)'\right)+F_{\left(\mu_2 RD^m_{\lambda,\alpha}f_2\right)'(U)}\left(\mu_2\left(RD^m_{\lambda,\alpha}f_2(z)\right)'\right)}{2}=$$

$$\frac{F_{\left(RD^m_{\lambda,\alpha}f_1\right)'(U)}\left(RD^m_{\lambda,\alpha}f_1(z)\right)'+F_{\left(RD^m_{\lambda,\alpha}f_2\right)'(U)}\left(RD^m_{\lambda,\alpha}f_2(z)\right)'}{2}.$$

Since $f_1,f_2\in RD^F_m(\delta,\lambda,\alpha)$ we have $\delta<F_{\left(RD^m_{\lambda,\alpha}f_1\right)'(U)}\left(RD^m_{\lambda,\alpha}f_1(z)\right)'\le 1$ and $\delta<F_{\left(RD^m_{\lambda,\alpha}f_2\right)'(U)}\left(RD^m_{\lambda,\alpha}f_2(z)\right)'\le 1$, $z\in U$.

Therefore $\delta<\dfrac{F_{\left(RD^m_{\lambda,\alpha}f_1\right)'(U)}\left(RD^m_{\lambda,\alpha}f_1(z)\right)'+F_{\left(RD^m_{\lambda,\alpha}f_2\right)'(U)}\left(RD^m_{\lambda,\alpha}f_2(z)\right)'}{2}\le 1$ and we obtain that $\delta<F_{\left(RD^m_{\lambda,\alpha}h\right)'(U)}\left(RD^m_{\lambda,\alpha}h(z)\right)'\le 1$, which means that $h\in RD^F_m(\delta,\lambda,\alpha)$ and $RD^F_m(\delta,\lambda,\alpha)$ is convex.

We highlight a fuzzy subset obtained using a convex function. Let the function $h(z)=\dfrac{1+z}{1-z}$, $z\in U$. After a short calculation we obtain that $Re\left(\dfrac{zh''(z)}{h'(z)}+1\right)=Re\dfrac{1+z}{1-z}>0$, so $h\in K$ and $h(U)=\{z\in C: Rez>0\}$. We define the membership function for the set $h(U)$ as $F_{h(U)}\left(h(z)\right)=Reh(z)$, $z\in U$ and we have $F_{h(U)}h(z)=\operatorname{supp}\left(h(U),F_{h(u)}\right)=\{z\in C:0<F_{h(U)}\left(h(z)\right)\le 1\}$ $=\{z\in U: 0<Rez\le 1\}$.

Theorem 2.2 Let g be a convex function in U and let $h(z)=g(z)+\dfrac{1}{c+2}zg'(z)$, where $z\in U$, $c>0$.

If $f\in RD^F_m(\delta,\lambda,\alpha)$ and $G(z)=I_c(f)(z)=\dfrac{c+2}{z^{c+1}}\int_0^z t^c f(t)\,dt$, $z\in U$, then;

$$F_{\left(RD^m_{\lambda,\alpha}f\right)'(U)}\left(RD^m_{\lambda,\alpha}f(z)\right)'\le F_{h(U)}h(z),\ \text{i.e.}\left(RD^m_{\lambda,\alpha}f(z)\right)'\prec_F h(z), z\in U, \tag{2.2}$$

implies $F_{\left(RD^m_{\lambda,\alpha}G\right)'(U)}\left(RD^m_{\lambda,\alpha}G(z)\right)'\le F_{g(U)}g(z)$, i.e. $\left(RD^m_{\lambda,\alpha}G(z)\right)'\prec_F g(z)$, $z\in U$, and this result is sharp.

Proof. We obtain that;

$$z^{c+1}G(z)=(c+2)\int_0^z t^c f(t)\,dt. \tag{2.3}$$

Differentiating (2.3), with respect to z, we have $(c+1)G(z)+zG'(z)=(c+2)f(z)$ and;

$$(c+1)RD^m_{\lambda,\alpha}G(z)+z\left(RD^m_{\lambda,\alpha}G(z)\right)'=(c+2)RD^m_{\lambda,\alpha}f(z),\ z\in U. \tag{2.4}$$

Differentiating (2.4) we have

$$\left(RD^m_{\lambda,\alpha}G(z)\right)'+\frac{1}{c+2}z\left(RD^m_{\lambda,\alpha}G(z)\right)''=\left(RD^m_{\lambda,\alpha}f(z)\right)', z\in U. \tag{2.5}$$

Using (2.5), the fuzzy differential subordination (2.2) becomes

$$\begin{aligned}&F_{RD^m_{\lambda,\alpha}G(U)}\left(\left(RD^m_{\lambda,\alpha}G(z)\right)'+\frac{1}{c+2}z\left(RD^m_{\lambda,\alpha}G(z)\right)''\right)\\&\le F_{g(U)}\left(g(z)+\frac{1}{c+2}zg'(z)\right).\end{aligned} \tag{2.6}$$

If we denote;

$$p(z)=\left(RD^m_{\lambda,\alpha}G(z)\right)', z\in U, \tag{2.7}$$

then $p\in H[1,1].$,

Replacing (2.7) in (2.6) we obtain $F_{p(U)}\left(p(z)+\dfrac{1}{c+2}zp'(z)\right)\le F_{g(U)}\left(g(z)+\dfrac{1}{c+2}zg'(z)\right)$, $z\in U$. Using Lemma 1.3 we have $F_{p(U)}p(z)\le F_{g(U)}g(z)$, $z\in U$, i.e. $F_{\left(RD^m_{\lambda,\alpha}G\right)'(U)}\left(RD^m_{\lambda,\alpha}G(z)\right)'\le F_{g(U)}g(z)$, $z\in U$, and g is the best dominant. We have obtained that $\left(RD^m_{\lambda,\alpha}G(z)\right)'\prec_F g(z)$, $z\in U$.

Theorem 2.3 Let $h(z)=\dfrac{1+(2\delta-1)z}{1+z}$, $\delta\in[0,1)$ and $c>0$. If $\alpha,\lambda\ge 0$, $m\in N$ and $I_c(f)(z)=\dfrac{c+2}{z^{c+1}}\int_0^z t^c f(t)\,dt$, $z\in U$, then;

$$I_c\left[RD^F_m(\delta,\lambda,\alpha)\right]\subset RD^F_m\left(\delta^*,\lambda,\alpha\right), \tag{2.8}$$

where $\delta^*=2\delta-1+(c+2)(2-2\delta)\beta(c)$ and $\beta(x)=\int_0^1\dfrac{t^{x+1}}{t+1}dt$.

Proof. The function h is convex and using the same steps as in the proof of Theorem 2.2 we get from the hypothesis of Theorem 2.3 that $F_{p(U)}\left(p(z)+\dfrac{1}{c+2}zp'(z)\right)\le F_{h(U)}h(z)$, where $p(z)$ is defined in (2.7).

Using Lemma 1.2 we deduce that $F_{p(U)}p(z) \le F_{g(U)}g(z) \le F_{h(U)}h(z)$, i.e. $F_{\left(RD^m_{\lambda,\alpha}G\right)'(U)}\left(RD^m_{\lambda,\alpha}G(z)\right)' \le F_{g(U)}g(z) \le F_{h(U)}h(z)$, where $g(z) = \frac{c+2}{z^{c+2}}\int_0^z t^{c+1}\frac{1+(2\delta-1)t}{1+t}dt = (2\delta-1) + \frac{(c+2)(2-2\delta)}{z^{c+2}}\int_0^z \frac{t^{c+1}}{1+t}dt$. Since g is convex and $g(U)$ is symmetric with respect to the real axis, we deduce;

$$F_{\left(RD^m_{\lambda,\alpha}G\right)'(U)}\left(RD^m_{\lambda,\alpha}G(z)\right)' \ge \min_{|z|=1} F_{g(U)}g(z) = F_{g(U)}g(1) \tag{2.9}$$

and $\delta^* = g(1) = 2\delta - 1 + (c+2)(2-2\delta)\beta(c)$. From (2.9) we deduce inclusion (2.8).

Theorem 2.4 Let g be a convex function, $g(0)=1$ and let h be the function $h(z) = g(z) + zg'(z)$, $z \in U$. If $\alpha,\lambda \ge 0$, $m \in N$, $f \in A$ and satisfies the fuzzy differential subordination

$$F_{\left(RD^m_{\lambda,\alpha}f\right)'(U)}\left(RD^m_{\lambda,\alpha}f(z)\right)' \le F_{h(U)}h(z), \text{i.e.}\left(RD^m_{\lambda,\alpha}f(z)\right)' \prec_F h(z), z \in U, \tag{2.10}$$

then $F_{RD^m_{\lambda,\alpha}f(U)}\frac{RD^m_{\lambda,\alpha}f(z)}{z} \le F_{g(U)}g(z)$, i.e. $\frac{RD^m_{\lambda,\alpha}f(z)}{z} \prec_F g(z)$, $z \in U$, and this result is sharp.

Proof. By using the properties of operator $RD^m_{\lambda,\alpha}$, we have

$$RD^m_{\lambda,\alpha}f(z) = z + \sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^m + (1-\alpha)C^m_{m+j-1}\right\}a_j z^j,$$

$z \in U$.

Consider

$$p(z) = \frac{RD^m_{\lambda,\alpha}f(z)}{z} = \frac{z + \sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^m + (1-\alpha)C^m_{m+j-1}\right\}a_j z^j}{z}$$

$= 1 + p_1 z + p_2 z^2 + \ldots, z \in U$. We deduce that $p \in H[1,1]$.

Let $RD^m_{\lambda,\alpha}f(z) = zp(z)$, $z \in U$. Differentiating we obtain $\left(RD^m_{\lambda,\alpha}f(z)\right)' = p(z) + zp'(z)$, $z \in U$. Then (2.10) becomes $F_{p(U)}\left(p(z)+zp'(z)\right) \le F_{h(U)}h(z) = F_{g(U)}\left(g(z)+zg'(z)\right)$, $z \in U$.

By using Lemma 1.3, we have $F_{p(U)}p(z) \le F_{g(U)}g(z)$, $z \in U$, i.e. $F_{RD^m_{\lambda,\alpha}f(U)}\frac{RD^m_{\lambda,\alpha}f(z)}{z} \le F_{g(U)}g(z)$, $z \in U$. We obtained that $\left(RD^m_{\lambda,\alpha}f(z)\right)' \prec_F h(z)$, $z \in U$, and this results is sharp.

Theorem 2.5 Let h be an holomorphic function which satisfies the inequality $\operatorname{Re}\left(1+\frac{zh''(z)}{h'(z)}\right) > -\frac{1}{2}$, $z \in U$, and $h(0)=1$. If $\alpha,\lambda \ge 0$, $m \in N$, $f \in A$ and satisfies the fuzzy differential subordination

$$F_{\left(RD^m_{\lambda,\alpha}f\right)'(U)}\left(RD^m_{\lambda,\alpha}f(z)\right)' \le F_{h(U)}h(z), \text{i.e.}\left(RD^m_{\lambda,\alpha}f(z)\right)' \prec_F h(z), z \in U, \tag{2.11}$$

then $F_{RD^m_{\lambda,\alpha}f(U)}\frac{RD^m_{\lambda,\alpha}f(z)}{z} \le F_{q(U)}q(z)$, i.e. $\frac{RD^m_{\lambda,\alpha}f(z)}{z} \prec_F q(z)$, $z \in U$, where $q(z) = \frac{1}{z}\int_0^z h(t)dt$. The function q is convex and it is the fuzzy best dominant.

Proof. Let

$$p(z) = \frac{RD^m_{\lambda,\alpha}f(z)}{z} = \frac{z + \sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^m + (1-\alpha)C^m_{m+j-1}\right\}a_j z^j}{z} =$$

$$1 + \sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^m + (1-\alpha)C^m_{m+j-1}\right\}a_j z^{j-1} =$$

$$1 + \sum_{j=2}^{\infty}p_j z^{j-1}, \quad z \in U, \quad p \in H[1,1].$$

Since $\operatorname{Re}\left(1+\frac{zh''(z)}{h'(z)}\right) > -\frac{1}{2}$, $z \in U$, from Lemma 1.1, we obtain that $q(z) = \frac{1}{z}\int_0^z h(t)dt$ is a convex function and verifies the differential equation asscociated to the fuzzy differential subordination (2.11) $q(z) + zq'(z) = h(z)$, therefore it is the fuzzy best dominant.

Differentiating, we obtain $\left(RD^m_{\lambda,\alpha}f(z)\right)' = p(z) + zp'(z)$, for $z \in U$ and (2.11) becomes $F_{p(U)}\left(p(z)+zp'(z)\right) \le F_{h(U)}h(z)$, $z \in U$. Using Lemma 1.2, we have $F_{p(U)}p(z) \le F_{q(U)}q(z)$, $z \in U$, i.e. $F_{RD^m_{\lambda,\alpha}f(U)}\frac{RD^m_{\lambda,\alpha}f(z)}{z} \le F_{q(U)}q(z)$, $z \in U$. We have obtained that $\frac{RD^m_{\lambda,\alpha}f(z)}{z} \prec_F q(z)$, $z \in U$.

Corollary 2.6 Let $h(z)=\frac{1+(2\beta-1)z}{1+z}$ a convex function in U, $0\le\beta<1$. If $\alpha,\lambda\ge 0$, $m\in N$, $f\in A$ and satisfies the fuzzy differential subordination;

$$F_{\left(RD^m_{\lambda,\alpha}f\right)'(U)}\left(RD^m_{\lambda,\alpha}f(z)\right)'\le F_{h(U)}h(z),\text{i.e.}\left(RD^m_{\lambda,\alpha}f(z)\right)'\prec_F h(z), z\in U, \tag{2.12}$$

then $F_{RD^m_{\lambda,\alpha}f(U)}\frac{RD^m_{\lambda,\alpha}f(z)}{z}\le F_{q(U)}q(z)$, i.e. $\frac{RD^m_{\lambda,\alpha}f(z)}{z}\prec_F q(z)$, $z\in U$, where q is given by $q(z)=2\beta-1+\frac{2(1-\beta)}{z}\ln(1+z)$, $z\in U$. The function q is convex and it is the fuzzy best dominant.

Proof. We have $h(z)=\frac{1+(2\beta-1)z}{1+z}$ with $h(0)=1$, $h'(z)=\frac{-2(1-\beta)}{(1+z)^2}$ and $h''(z)=\frac{4(1-\beta)}{(1+z)^3}$, therefore $Re\left(\frac{zh''(z)}{h'(z)}+1\right)=Re\left(\frac{1-z}{1+z}\right)=Re\left(\frac{1-\rho\cos\theta-i\rho\sin\theta}{1+\rho\cos\theta+i\rho\sin\theta}\right)=\frac{1-\rho^2}{1+2\rho\cos\theta+\rho^2}>0>-\frac{1}{2}$.

Following the same steps as in the proof of Theorem 2.5 and considering $p(z)=\frac{RD^m_{\lambda,\alpha}f(z)}{z}$, the differential subordination (2.12) becomes $F_{RD^m_{\lambda,\alpha}f(U)}\left(p(z)+zp'(z)\right)\le F_{h(U)}h(z)$, $z\in U$. By using Lemma 1.2 for $\gamma=1$, we have $F_{p(U)}p(z)\le F_{q(U)}q(z)$, i.e. $F_{RD^m_{\lambda,\alpha}f(U)}\frac{RD^m_{\lambda,\alpha}f(z)}{z}\le F_{q(U)}q(z)$ and $q(z)=\frac{1}{z}\int_0^z h(t)dt=\frac{1}{z}\int_0^z\frac{1+(2\beta-1)t}{1+t}dt=2\beta-1+\frac{2(1-\beta)}{z}\ln(1+z)$, $z\in U$.

Example 2.1 Let $h(z)=\frac{1-z}{1+z}$ a convex function in U with $h(0)=1$ and $Re\left(\frac{zh''(z)}{h'(z)}+1\right)>-\frac{1}{2}$.

Let $f(z)=z+z^2$, $z\in U$. For $m=1$, $\lambda=\frac{1}{2}$, $\alpha=2$, we obtain $RD^1_{\frac{1}{2},2}f(z)=-R^1f(z)+2D^1_{\frac{1}{2}}f(z)=-zf'(z)+2\left(\frac{1}{2}f(z)+\frac{1}{2}zf'(z)\right)=f(z)=z+z^2$, $z\in U$. Then $\left(RD^1_{\frac{1}{2},2}f(z)\right)'=f'(z)=1+2z$ and $\frac{RD^1_{\frac{1}{2},2}f(z)}{z}=1+z$. We have $q(z)=\frac{1}{z}\int_0^z\frac{1-t}{1+t}dt=-1+\frac{2\ln(1+z)}{z}$.

Using Theorem 2.5 we obtain $1+2z\prec_F\frac{1-z}{1+z}$, $z\in U$, induce $1+z\prec_F -1+\frac{2\ln(1+z)}{z}$, $z\in U$.

Theorem 2.7 Let g be a convex function such that $g(0)=1$ and let h be the function $h(z)=g(z)+zg'(z)$, $z\in U$. If $\alpha,\lambda\ge 0$, $m\in N$, $f\in A$ and the fuzzy differential subordination $F_{RD^m_{\lambda,\alpha}f(U)}\Big(\frac{(m+1)(m+2)}{z}RD^{m+2}_{\lambda,\alpha}f(z)-\frac{(m+1)(2m+1)}{z}RD^{m+1}_{\lambda,\alpha}f(z)+\frac{m^2}{z}RD^m_{\lambda,\alpha}f(z)-\frac{\alpha\left[(m+1)(m+2)-\frac{1}{\lambda^2}\right]}{z}D^{m+2}_{\lambda}f(z)+\frac{\alpha\left[(m+1)(2m+1)-\frac{2(1-\lambda)}{\lambda^2}\right]}{z}D^{m+1}_{\lambda}f(z)-\frac{\alpha\left[m^2-\frac{(1-\lambda)^2}{\lambda^2}\right]}{z}D^m_{\lambda}f(z)\Big)\le F_{h(U)}h(z)$, i.e.

$$\frac{(m+1)(m+2)}{z}RD^{m+2}_{\lambda,\alpha}f(z)-\frac{(m+1)(2m+1)}{z}RD^{m+1}_{\lambda,\alpha}f(z)+\frac{m^2}{z}RD^m_{\lambda,\alpha}f(z)-\frac{\alpha\left[(m+1)(m+2)-\frac{1}{\lambda^2}\right]}{z}D^{m+2}_{\lambda}f(z)+\frac{\alpha\left[(m+1)(2m+1)-\frac{2(1-\lambda)}{\lambda^2}\right]}{z}D^{m+1}_{\lambda}f(z)-\frac{\alpha\left[m^2-\frac{(1-\lambda)^2}{\lambda^2}\right]}{z}D^m_{\lambda}f(z)\prec_F h(z),\quad z\in U, \tag{2.13}$$

holds, then $F_{\left(RD^m_{\lambda,\alpha}f\right)'(U)}[RD^m_{\lambda,\alpha}f(z)]'\le F_{g(U)}g(z)$, i.e. $[RD^m_{\lambda,\alpha}f(z)]'\prec_F g(z)$, $z\in U$. This result is sharp.

Proof. Let

$$p(z)=\left(RD^m_{\lambda,\alpha}f(z)\right)'=(1-\alpha)\left(R^mf(z)\right)'+\alpha\left(D^m_\lambda f(z)\right)' \tag{2.14}$$

$$=1+\sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^m+(1-\alpha)C^m_{m+j-1}\right\}ja_jz^{j-1}$$

$=1+p_1z+p_2z^2+\ldots$. We deduce that $p\in H[1,1]$.

By using the properties of operators $RD^m_{\lambda,\alpha}$, R^m and D^m_λ, after a short calculation, we obtain;

$$p(z)+zp^{'}(z)=\frac{(m+1)(m+2)}{z}RD_{\lambda,\alpha}^{m+2}f(z)-\frac{(m+1)(2m+1)}{z}RD_{\lambda,\alpha}^{m+1}f(z)+\frac{m^2}{z}RD_{\lambda,\alpha}^{m}f(z)-\frac{\alpha\left[(m+1)(m+2)-\frac{1}{\lambda^2}\right]}{z}D_{\lambda}^{m+2}f(z)+\frac{\alpha\left[(m+1)(2m+1)-\frac{2(1-\lambda)}{\lambda^2}\right]}{z}D_{\lambda}^{m+1}f(z)-\frac{\alpha\left[m^2-\frac{(1-\lambda)^2}{\lambda^2}\right]}{z}D_{\lambda}^{m}f(z).$$

Using the notation in (2.14), the fuzzy differential subordination becomes $F_{p(U)}\left(p(z)+zp^{'}(z)\right)\le F_{h(U)}h(z)=F_{g(U)}\left(g(z)+zg^{'}(z)\right)$. By using Lemma 1.3, we have $F_{p(U)}p(z)\le F_{g(U)}g(z),\quad z\in U$, i.e. $F_{RD_{\lambda,\alpha}^m f(U)}\left(RD_{\lambda,\alpha}^m f(z)\right)^{'}\le F_{g(U)}g(z),\quad z\in U$, and this result is sharp.

Theorem 2.8 Let h be an holomorphic function which satisfies the inequality $\operatorname{Re}\left[1+\frac{zh^{''}(z)}{h^{'}(z)}\right]>-\frac{1}{2}$, $z\in U$, and $h(0)=1$. If $\alpha,\lambda\ge 0$, $m\in N$, $f\in A$ and satisfies the fuzzy differential subordination;

$$F_{RD_{\lambda,\alpha}^m f(U)}\Big(\frac{(m+1)(m+2)}{z}RD_{\lambda,\alpha}^{m+2}f(z)-\frac{(m+1)(2m+1)}{z}RD_{\lambda,\alpha}^{m+1}f(z)+\frac{m^2}{z}RD_{\lambda,\alpha}^{m}f(z)-\frac{\alpha\left[(m+1)(m+2)-\frac{1}{\lambda^2}\right]}{z}D_{\lambda}^{m+2}f(z)+\frac{\alpha\left[(m+1)(2m+1)-\frac{2(1-\lambda)}{\lambda^2}\right]}{z}D_{\lambda}^{m+1}f(z)-\frac{\alpha\left[m^2-\frac{(1-\lambda)^2}{\lambda^2}\right]}{z}D_{\lambda}^{m}f(z)\Big)\le F_{h(U)}h(z)$$, i.e.

$$\frac{(m+1)(m+2)}{z}RD_{\lambda,\alpha}^{m+2}f(z)-\frac{(m+1)(2m+1)}{z}RD_{\lambda,\alpha}^{m+1}f(z)+\frac{m^2}{z}RD_{\lambda,\alpha}^{m}f(z)-\frac{\alpha\left[(m+1)(m+2)-\frac{1}{\lambda^2}\right]}{z}D_{\lambda}^{m+2}f(z)+\frac{\alpha\left[(m+1)(2m+1)-\frac{2(1-\lambda)}{\lambda^2}\right]}{z}D_{\lambda}^{m+1}f(z)-\frac{\alpha\left[m^2-\frac{(1-\lambda)^2}{\lambda^2}\right]}{z}D_{\lambda}^{m}f(z)\prec_F h(z),\quad z\in U,\tag{2.15}$$

then $F_{RD_{\lambda,\alpha}^m f(U)}\left(RD_{\lambda,\alpha}^m f(z)\right)^{'}\le F_{q(U)}q(z)$.i.e. $\left(RD_{\lambda,\alpha}^m f(z)\right)^{'}\prec_F q(z),\quad z\in U$, where q is given by $q(z)=\frac{1}{z}\int_0^z h(t)dt$. The function q is convex and it is the fuzzy best dominant.

Proof. Since $\operatorname{Re}\left(1+\frac{zh^{''}(z)}{h^{'}(z)}\right)>-\frac{1}{2},\quad z\in U$, from Lemma 1.1, we obtain that $q(z)=\frac{1}{z}\int_0^z h(t)dt$ is a convex function and verifies the differential equation asscociated to the fuzzy differential subordination (2.11) $q(z)+zq^{'}(z)=h(z)$, therefore it is the fuzzy best dominant.

Using the properties of operator $RD_{\lambda,\alpha}^m$ and considering $p(z)=\left(RD_{\lambda,\alpha}^m f(z)\right)^{'}$, we obtain

$$p(z)+zp^{'}(z)=\frac{(m+1)(m+2)}{z}RD_{\lambda,\alpha}^{m+2}f(z)-\frac{(m+1)(2m+1)}{z}RD_{\lambda,\alpha}^{m+1}f(z)+\frac{m^2}{z}RD_{\lambda,\alpha}^{n}f(z)-\frac{\alpha\left[(m+1)(m+2)-\frac{1}{\lambda^2}\right]}{z}D_{\lambda}^{m+2}f(z)+\frac{\alpha\left[(m+1)(2m+1)-\frac{2(1-\lambda)}{\lambda^2}\right]}{z}D_{\lambda}^{m+1}f(z)-\frac{\alpha\left[m^2-\frac{(1-\lambda)^2}{\lambda^2}\right]}{z}D_{\lambda}^{m}f(z),\quad z\in U.$$

Then (2.15) becomes $F_{p(U)}\left(p(z)+zp^{'}(z)\right)\le F_{h(U)}h(z)$, $z\in U$. Since $p\in H[1,1]$, using Lemma 1.2, we deduce $F_{p(U)}p(z)\le F_{q(U)}q(z),\ z\in U$, i.e. $F_{RD_{\lambda,\alpha}^m f(U)}\left(RD_{\lambda,\alpha}^m f(z)\right)^{'}\le F_{q(U)}q(z)$, $z\in U$.

Corollary 2.9 Let $h(z)=\frac{1+(2\beta-1)z}{1+z}$ be a convex function in U, where $0\le\beta<1$. If $\alpha,\lambda\ge 0$, $m\in N$, $f\in A$ and satisfies the fuzzy differential subordination

$$F_{RD_{\lambda,\alpha}^m f(U)}\Big(\frac{(m+1)(m+2)}{z}RD_{\lambda,\alpha}^{m+2}f(z)-\frac{(m+1)(2m+1)}{z}RD_{\lambda,\alpha}^{m+1}f(z)+\frac{m^2}{z}RD_{\lambda,\alpha}^{m}f(z)-\frac{\alpha\left[(m+1)(m+2)-\frac{1}{\lambda^2}\right]}{z}D_{\lambda}^{m+2}f(z)$$

$$+\frac{\alpha\left[(m+1)(2m+1)-\frac{2(1-\lambda)}{\lambda^2}\right]}{z}D_\lambda^{m+1}f(z)-\frac{\alpha\left[m^2-\frac{(1-\lambda)^2}{\lambda^2}\right]}{z}D_\lambda^m f(z))$$

$\le F_{h(U)}h(z)$, i.e.

$$\frac{(m+1)(m+2)}{z}RD_{\lambda,\alpha}^{m+2}f(z)-\frac{(m+1)(2m+1)}{z}RD_{\lambda,\alpha}^{m+1}f(z)+\frac{m^2}{z}RD_{\lambda,\alpha}^m f(z)-\frac{\alpha\left[(m+1)(m+2)-\frac{1}{\lambda^2}\right]}{z}D_\lambda^{m+2}f(z)+\frac{\alpha\left[(m+1)(2m+1)-\frac{2(1-\lambda)}{\lambda^2}\right]}{z}D_\lambda^{m+1}f(z)-\frac{\alpha\left[m^2-\frac{(1-\lambda)^2}{\lambda^2}\right]}{z}D_\lambda^m f(z)\prec_F h(z),\quad z\in U, \tag{2.16}$$

then $F_{RD_{\lambda,\alpha}^m f(U)}\left(RD_{\lambda,\alpha}^m f(z)\right)'\le F_{q(U)}q(z)$, i.e. $\left(RD_{\lambda,\alpha}^m f(z)\right)'\prec_F q(z)$, $z\in U$, where q is given by $q(z)=2\beta-1+\frac{2(1-\beta)}{z}\ln(1+z)$, $z\in U$. The function q is convex and it is the fuzzy best dominant.

Proof. Following the same steps as in the proof of Theorem 2.7 and considering $p(z)=\left(RD_{\lambda,\alpha}^m f(z)\right)'$, the differential subordination (2.16) becomes $F_{p(U)}\left(p(z)+zp'(z)\right)\le F_{h(U)}h(z)$, $z\in U$. By using Lemma 1.2 for $\gamma=1$, we have $F_{p(U)}p(z)\le F_{q(U)}q(z)$, i.e. $F_{\left(RD_{\lambda,\alpha}^m f\right)'(U)}\left(RD_{\lambda,\alpha}^m f(z)\right)'\le F_{q(U)}q(z)$, i.e. $\left(RD_{\lambda,\alpha}^m f(z)\right)'\prec_F q(z)$, and $q(z)=\frac{1}{z}\int_0^z h(t)dt=\frac{1}{z}\int_0^z\frac{1+(2\beta-1)t}{1+t}dt=2\beta-1+\frac{2(1-\beta)}{z}\ln(1+z)$, $z\in U$.

Example 2.2 Let $h(z)=\frac{1-z}{1+z}$ a convex function in U with $h(0)=1$ and $Re\left(\frac{zh''(z)}{h'(z)}+1\right)>-\frac{1}{2}$.

Let $f(z)=z+z^2$, $z\in U$. For $m=1$, $\lambda=\frac{1}{2}$, $\alpha=2$, we obtain $RD_{\frac{1}{2},2}^1 f(z)=-R^1 f(z)+2D_{\frac{1}{2}}^1 f(z)=-zf'(z)+2\left(\frac{1}{2}f(z)+\frac{1}{2}zf'(z)\right)=f(z)=z+z^2$ and

$(m+1)RD_{\lambda,\alpha}^{m+1}f(z)-(m-1)RD_{\lambda,\alpha}^m f(z)\quad -\alpha\left(m+1-\frac{1}{\lambda}\right)\left[D_\lambda^{m+1}f(z)-D_\lambda^m f(z)\right]=2RD_{\frac{1}{2},2}^2 f(z)=-2+2z$, where $RD_{\frac{1}{2},2}^2 f(z)=-R^2 f(z)+2D_{\frac{1}{2}}^2 f(z)=-\left(1+3z^2\right)+2\left(\frac{1}{2}z+\frac{3}{2}z^2\right)=-1+z$. We have $q(z)=\frac{1}{z}\int_0^z\frac{1-t}{1+t}dt=-1+\frac{2\ln(1+z)}{z}$.

Using Theorem 2.8 we obtain $-2+2z\prec_F\frac{1-z}{1+z}$, $z\in U$, induce $z+z^2\prec_F -1+\frac{2\ln(1+z)}{z}$, $z\in U$.

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