Optical Soliton Solutions of Fokas-Lenells Equation via $\left(m+\frac{1}{G'}\right)$-Expansion Method

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Abstract: In this research paper, we investigate some novel soliton solutions to the perturbed Fokas-Lenells equation by using the $\left(m+\frac{1}{G'}\right)$-expansion method. Some new solutions are obtained and they are plotted in two and three dimensions. This technique appears as a suitable, applicable, and efficient method to search for the exact solutions of nonlinear partial differential equations in a wide range. All gained optical soliton solutions are substituted into the Fokas-Lenells equation and they verify it. The constraint conditions are also given.

Keywords: Optical soliton solutions; $\left(m+\frac{1}{G'}\right)$-expansion method; Fokas-Lenells equation.

1. INTRODUCTION

The study of soliton solutions and its application has been noticeable in recent decades. New achievements of optical soliton solutions and their applications in the study of topological solitons and transformation phenolmena in polyacetylene chains with the action of an electrical field are noted continuously [1]. The connection between nonlinear complex physical phenomena and nonlinear partial differential equations (NLPDEs) is involved in many fields of sciences such as plasma physics, optical fibers, nonlinear optics, fluid mechanics, biology, chemistry kinetics, geochemistry, engineer ing, and so on [2]. NLPDEs have different analytical approaches used to find analytical solutions [11-13,17-19]. Different analytical approaches used to find the soliton solutions of the Fokas-Lenells equation as well as thecoupled Fokas-Lenells equation like the $\left(-\phi(\xi)\right)$ function approach toobtain soliton solutions to perturbed Fokas-Lenells equation [3]. Bright, dark, and singular soliton solutions are obtained by applying the extended trial function approach to find the soliton solutions to the Fokas-Lenells equation [4]. Optical soliton solutions to the Fokas-Lenells equation in birefringent fibers via the application of an extended trial function method [5]. Darboux transformation with using a limiting process was used to study all kinds of one-soliton solutions, including the bright-dark soliton, the dark-anti-dark soliton, and the breather-like solutions [6]. In [7], the authors investigated the Fokas-Lenells equation describing the propagation of ultra-short pulses in optical fibers. In [8], via Darboux transformation, the general couplednonlinear Fokas-Lenells system was studied. Optical soliton perturbation with Fokas-Lenells equation using three exotic and efficient integration schemes were investigated in [9]. Optical soliton perturbation with Fokas-Lenells equation by mapping methods were constructed in [10].

In this paper, we study some new soliton solutions of the Fokas-Lenells equation by using the $\left(m+\frac{1}{G'}\right)$-expansion method. The variable approach of the traveling wave will convert the NLPDEs tononlinear ordinary differential equations and we solvethis equation analytically for different values of nonzero-constants.

1. The $\left(m+\frac{1}{G'}\right)$-Expansion Method

The mainly modified steps of these techniques can be taken as follows [16]:

***Step* 1**: Assuming a nonlinear partial differential equation (NLPDE) as follows:

$$P\left(u,u_x,u_t,uu_x,\ \ldots\right)=0, \tag{1}$$

and using the traveling wave transformation,

$$\phi\left(x,t\right)=U\left(\xi\right),\ \xi=\alpha x\quad \beta t. \tag{2}$$

Using Eq. (2) to Eq. (1) yields a nonlinear ordinary differential equation (NLODE) as following:

$$N\left(U,U',U'',\ \cdots\right)=0. \tag{3}$$

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E-ISSN: 2409-5761/20

***Step* 2**:Take trial equation of solution for Eq. (3) as following:

$$U(\xi) = \sum_{i=-n}^{n} \rho_i (m+F)^i = m\rho_0 + \rho_1 (m+F) + \rho_2 (m+F)^2 + \ldots + \rho_n (m+F)^n, \tag{4}$$

where $\rho_n,\ n = 0,1,\ldots,n$ and m are nonzero constants. According to the principles of balance, we find the value of n. In this manuscript, we let

$$F = \frac{1}{G'}, \tag{5}$$

where $G(\xi)$ verify $G'' + (\lambda + 2m\mu)G' + \mu = 0.$

***Step* 3**: Putting the Eq. (4) into Eq. (3) and using Eqs. (5), then collecting all terms with the same order of the $(m+F)^n$, we get the system of algebraic equations for $\rho_n,\ n = 0,1,\ldots,n$, α and β.

***Step* 4**: As a result, we solved the obtained system and substituted α, β, $\rho_n,\ n = 0,1,\ldots,n$ and inserted a general solution of the LODE Eq. (5) into Eq. (4) to get the explicit and exact solution of Eq. (1).

2. Governing Model and Mathematical Analysis

The Fokas-Lenells equation (FLE) besides its complex terms in non-dimension form is [14-15]:

$$iu_t + a_1 u_{xx} + a_2 u_{xt}(x,t) + \left|u(x,t)\right|^2 \begin{pmatrix} bu(x,t) + \\ i\sigma u_x(x,t) \end{pmatrix} = i\begin{pmatrix} \alpha u_x(x,t) + \gamma\left(\left|u(x,t)\right|^{2m} u(x,t)\right)x + \\ \delta\left(\left|u(x,t)\right|^{2m}\right)_x u(x,t) \end{pmatrix}. \tag{6}$$

Setting $m = 1$, we get

$$iu_t + a_1 u_{xx} + a_2 u_{xt}(x,t) + \left|u(x,t)\right|^2 \begin{pmatrix} bu(x,t) + \\ i\sigma u_x(x,t) \end{pmatrix} = i\begin{pmatrix} \alpha u_x(x,t) + \gamma\left(\left|u(x,t)\right|^{2} u(x,t)\right)x + \\ \delta\left(\left|u(x,t)\right|^{2}\right)_x u(x,t) \end{pmatrix}, \tag{7}$$

here $u(x,t)$ symbolizes a complex field envelope, x, t symbolizes spatial and temporal variable. the coefficients a_1, b represent velocity dispersion and a_2 is spatiotemporal dispersion while σ and δ are the nonlinear dispersion term that provides the additional dispersive effect. Moreover, α provides the inter-modal dispersion, γ is the self-steepening term that prevents the formation of shock waves. The complexity terms with γ and δ comes with full nonlinearity where m provides the full nonlinearity parameter.

To start and apply the sine-Gordon expansion method, we let wave transform as

$$u(x,t) = U(\xi)e^{i\theta(x,t)},\ \xi = x - \upsilon t, \tag{8}$$

$$\theta(x,t) = -kx + \omega t + \theta_0. \tag{9}$$

Where $U(\xi)$ and $\theta(x,t)$symbolize the shape of the pulse and the phase component, respectively. υ, k, ω and θ_0 represents the velocity of the soliton, the frequency, the soliton wave number, and phase constant, respectively. Using (8) and (9) with (7) and separating the minto imaginary and real parts, the following pair of equations respectively gives

$$(\upsilon a_2 k + a_2\omega - \upsilon - \alpha - 2a_1 k)U' + (\sigma - 3\gamma - 2\delta)U^2 U' = 0 \tag{10}$$

Equaling the linear coefficients of independent functions to zero gives

$$\upsilon = \frac{a_2\omega - \alpha - 2a_1 k}{a_2 k - 1}, \tag{11}$$

$$\sigma = 3\gamma + 2\delta. \tag{12}$$

These symbolize the speed of the wave and a couple of constraint relations on the parameters as well. On the other side, the real part yields

$$(a_1 - \upsilon a_2)U'' - (w + a_1 k^2 - a_2 k\omega + \alpha k)U + (b + \sigma k - \gamma k)U^3 = 0 \tag{13}$$

After evaluating the balance between the highest order and the highest power of nonlinear terms of Eq. (13), we obtain $n = 1$.

Using the value of balance and putting it into Eq. (4), we obtain

$$U(\xi) = \rho_{-1}(m+F)^{-1} + m\rho_0 + \rho_1(m+F), \tag{14}$$

and

$$F = \frac{1}{G'} \tag{15}$$

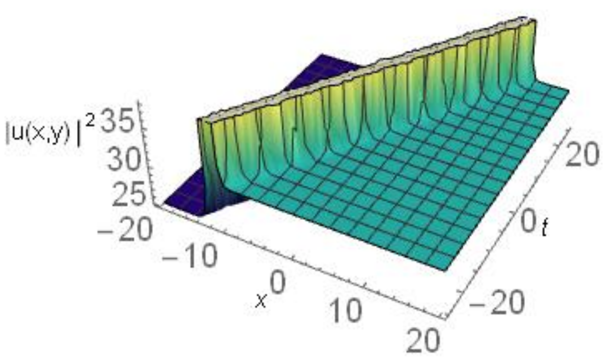
Using Eq. (14) along with Eq. (15), and implementting it to Eq. (13), we get a trigonometric equation. Solving the resultant equation, we discuss the following cases:

Set1: when we choose

$$\alpha = -\frac{a_1}{a_2}, \omega = -\frac{a_1 b}{2a_2(\gamma+\delta)}, k = \frac{b}{2(\gamma+\delta)},$$

we obtain

$$u_1 = e^{\frac{1}{2}i\left(\frac{b(-a_1 t + a_2 x)}{a_2(\gamma+\delta)} + 2\theta_0\right)} \left(\rho_{-1}\left(m + \frac{1}{A_1 e^{\frac{(a_1 t - a_2 x)(\lambda + 2m\mu)}{a2}} - \frac{\mu}{\lambda+2m\mu}} \right)^{-1} + \rho_0 + \left(m + \frac{1}{A_1 e^{\frac{(a_1 t - a_2 x)(\lambda + 2m\mu)}{a2}} - \frac{\mu}{\lambda+2m\mu}} \right)\rho_1 \right) \tag{16}$$



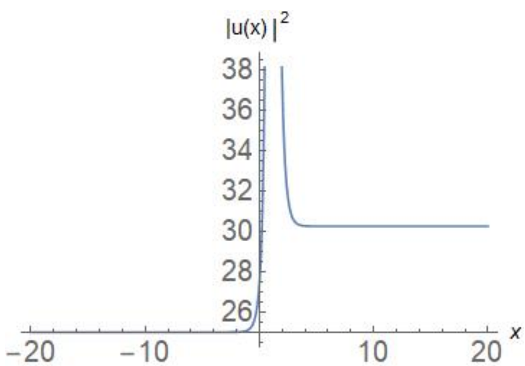


Figure 1 (contd….)

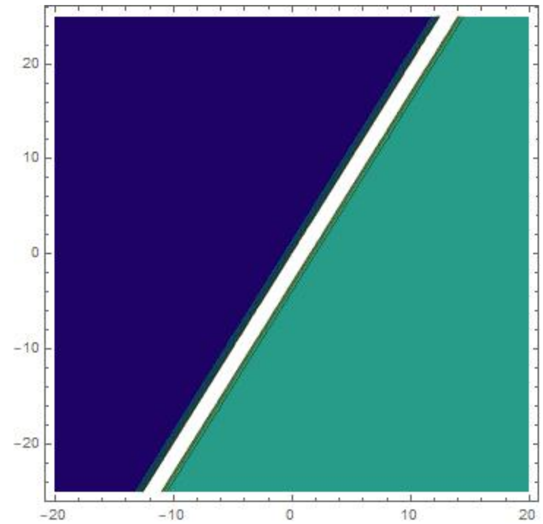


Figure 1: The optical solitonsolutionsof 16), when $A_1 = -2, a_1 = 1, a_2 = 2, \gamma = 3, \delta = 2, b = 1, \rho_0 = 1, \rho_1 = 1, \rho_{-1} = 3, \lambda = -1, m = 1, \mu = 2$ and for 2-D $t = 2$.

Set 2: when we choose

$$\rho_{-1} = 0, \alpha = -\frac{a_1}{a_2}, \omega = -\frac{a_1 b}{2a_2(\gamma+\delta)}, k = \frac{b}{2(\gamma+\delta)},$$

we gain

$$u_2(x,t) = e^{\frac{1}{2}i\left(\frac{b(a_2 x - a_1 t)}{a_2(\gamma+\delta)} + 2\theta_0\right)} \left(\rho_0 + \rho_1 \left(m + \frac{1}{A_1 e^{\frac{(a_1 t - a_2 x)(\lambda + 2m\mu)}{a2}} - \frac{\mu}{\lambda+2m\mu}} \right) \right) \tag{17}$$

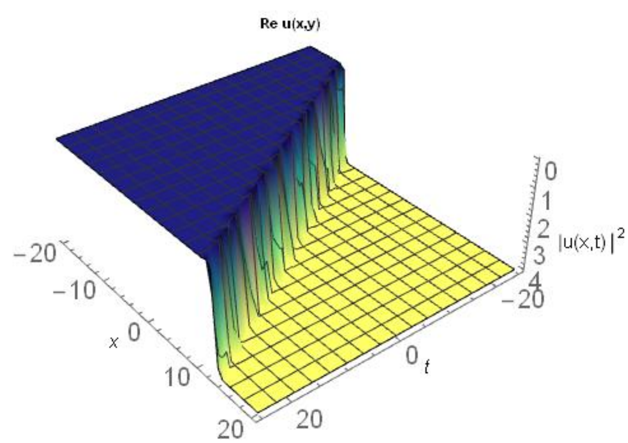


Figure 2 (contd….)

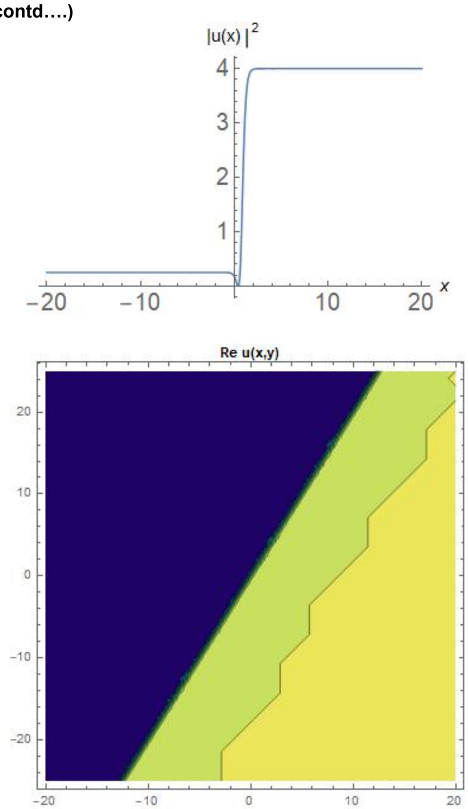


Figure 2: The optical solitonsolutions of (17), when $A_1=-2, a_1=1, a_2=2, \gamma=3, \delta=2, b=1, \rho_0=1, \rho_1=1, \rho_{-1}=3, \lambda=-1, m=1, \mu=-2$ and for 2-D $t=2$.

Set 3: When we choose

$$\alpha=-\frac{a_1\lambda^2+\left(b+2k\left(\gamma+\delta\right)\right)\left(2+a_2\left(-4k+2a_2k^2-a_2\left(\lambda+2m\mu\right)^2\right)\right)\rho_0^2}{a_2\lambda^2},$$

$$\rho_{-1}=-\frac{2m\left(\lambda+m\mu\right)\rho_0}{\lambda}, \rho_1=0, \omega=\frac{a_1k\lambda^2-\left(b+2k\left(\gamma+\delta\right)\right)\left(-2k+2a_2k^2-a_2\left(\lambda+2m\mu\right)^2\right)\rho_0^2}{a_2\lambda^2},$$

we get

$$u_3=\rho_0 e^{i\left(\frac{a_1kt}{a_2}-kx-\frac{t\left(b+2k\left(\gamma+\delta\right)\right)\left(-2k+2a_2k^2-a_2\left(\lambda+2m\mu\right)^2\right)\rho_0^2}{a_2\lambda^2}+\theta_0\right)}\left(1-\frac{2m\left(\lambda+m\mu\right)}{\lambda}\left(m+\frac{1}{A_1 e^{\frac{\left(\lambda+2m\mu\right)\left(\left(a_1t-a_2x\right)\lambda^2+2t\left(b+2k\left(\gamma+\delta\right)\right)\rho_0^2\right)}{a_2\lambda^2}}-\frac{\mu}{\lambda+2m\mu}}\right)^{-1}\right) \quad (18)$$

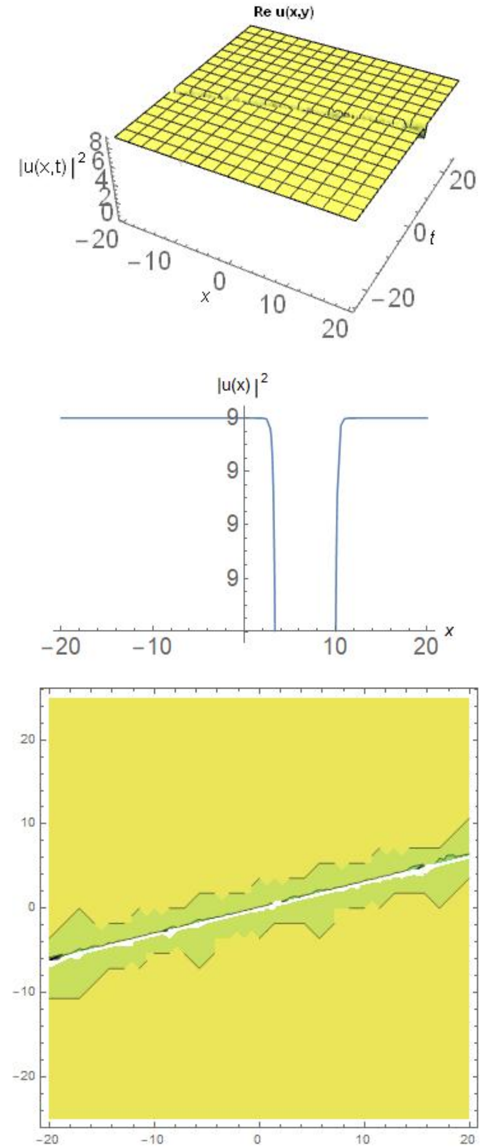


Figure 3: The optical solitonsolutions of (18), when $A_1=2, a_1=1, a_2=2, \gamma=3, \delta=2, b=1, \rho_0=1, \rho_1=1, \rho_{-1}=3, \lambda=2, m=1, \mu=2, k=1$ and for 2-D $t=2$.

CONCLUSION

In this manuscript, via the $\left(m+\frac{1}{G'}\right)$-expansion method the optical soliton solutions of perturbed

Fokas–Lenells equation with Kerr law nonlinearity are constructed and the obtained results satisfy the eq. (1). Based on the results, the suggested method is useful, efficient, and replicable for taking out the soliton solutions of strong nonlinear partial differential equations. The Mathematica software to find the exact solutions and plotting them in two- and three-dimensions are utilized. The results obtained in this research may be available with time in some sciences like applied mathematics, physics, and engineering.

REFERENCES

[1] T. Aktürk, Y. Gürefe, and H. Bulut, "New function method to the (n+1)-dimensional nonlinear problems," *An Int. J. Optim. Control Theor. Appl.*, 2017. https://doi.org/10.11121/ijocta.01.2017.00489

[2] O. A. Ilhan, A. Esen, H. Bulut, and H. M. Baskonus, "Singular solitons in the pseudo-parabolic model arising in nonlinear surface waves," *Results Phys.*, 2019. https://doi.org/10.1016/j.rinp.2019.01.059

[3] S. Arshed *et al.*, "Optical solitons pertutabation with Fokas-Lenells equation by exp(−ϕ(ξ))-expansion method," *Optik (Stuttg).*, 2019. https://doi.org/10.1016/j.ijleo.2018.10.136

[4] A. Biswas, M. Ekici, A. Sonmezoglu, and R. T. Alqahtani, "Optical soliton perturbation with full nonlinearity for Fokas–Lenells equation," *Optik (Stuttg).*, 2018. https://doi.org/10.1016/j.ijleo.2018.03.094

[5] A. Biswas, M. Ekici, A. Sonmezoglu, and R. T. Alqahtani, "Optical solitons with differential group delay for coupled Fokas–Lenells equation by extended trial function scheme," *Optik (Stuttg).*, 2018. https://doi.org/10.1016/j.ijleo.2018.03.102

[6] L. Ling, B. F. Feng, and Z. Zhu, "General soliton solutions to a coupled Fokas–Lenells equation," *Nonlinear Anal. Real World Appl.*, 2018. https://doi.org/10.1016/j.nonrwa.2017.08.013

[7] H. Triki and A. M. Wazwaz, "Combined optical solitary waves of the Fokas—Lenells equation," *Waves in Random and Complex Media*, 2017. https://doi.org/10.1080/17455030.2017.1285449

[8] Y. Zhang, J. W. Yang, K. W. Chow, and C. F. Wu, "Solitons, breathers and rogue waves for the coupled Fokas–Lenells system via Darboux transformation," *Nonlinear Anal. Real World Appl.*, 2017. https://doi.org/10.1016/j.nonrwa.2016.06.006

[9] A. Biswas *et al.*, "Optical soliton perturbation with Fokas–Lenells equation using three exotic and efficient integration schemes," *Optik (Stuttg).*, 2018. https://doi.org/10.1016/j.ijleo.2018.03.132

[10] E. V. Krishnan, A. Biswas, Q. Zhou, and M. Alfiras, "Optical soliton perturbation with Fokas–Lenells equation by mapping methods," *Optik (Stuttg).*, 2019. https://doi.org/10.1016/j.ijleo.2018.10.017

[11] H. S. Shukla, M. Tamsir, and V. K. Srivastava, "Numerical simulation of two dimensional sine-Gordon solitons using modified cubic B-spline differential quadrature method," *AIP Adv.*, 2015. https://doi.org/10.1063/1.4906256

[12] G. Yel, H. M. Baskonus, and H. Bulut, "Novel archetypes of new coupled Konno–Oono equation by using sine–Gordon expansion method," *Opt. Quantum Electron.*, 2017. https://doi.org/10.1007/s11082-017-1127-z

[13] H. M. Baskonus and H. Bulut, "New complex exact travelling wave solutions for the generalized-Zakharov equation with complex structures," *An Int. J. Optim. Control Theor. Appl.*, 2016. https://doi.org/10.11121/ijocta.01.2016.00295

[14] H. Triki and A. M. Wazwaz, "New types of chirped soliton solutions for the Fokas-Lenells equation," in *International Journal of Numerical Methods for Heat and Fluid Flow*, 2017. https://doi.org/10.1016/j.cnsns.2016.06.026

[15] J. He, S. Xu, and K. Porsezian, "Rogue waves of the fokas-lenells equation," *J. Phys. Soc. Japan*, 2012. https://doi.org/10.1143/JPSJ.81.124007

[16] Durur, H., Ilhan, E., & Bulut, H. (2020). Novel Complex Wave Solutions of the (2+ 1)- Dimensional Hyperbolic Nonlinear Schrödinger Equation. Fractal and Fractional, 4(3), 41. https://doi.org/10.3390/fractalfract4030041

[17] Yokus, A., Durur, H., Ahmad, H., & Yao, S. W. (2020). Construction of Different Types Analytic Solutions for the Zhiber-Shabat Equation. Mathematics, 8(6), 908. https://doi.org/10.3390/math8060908

[18] Durur, H. (2020). Different types analytic solutions of the (1+ 1)-dimensional resonant nonlinear Schrödinger's equation using (G′/G)-expansion method. Modern Physics Letters B, 34(03), 2050036. https://doi.org/10.1142/S0217984920500360

[19] Yokus, A., Durur, H., & Ahmad, H. (2020). Hyperbolic type solutions for the couple Boiti-Leon-Pempinelli system. FactaUniversitatis, Series: Mathematics and Informatics, 35(2), 523-531.

Received on 08-09-2020 Accepted on 03-10-2020 Published on 16-10-2020

DOI: https://doi.org/10.15377/2409-5761.2020.07.3