

Effects of Pressure Gradients on Energy Dissipation Coefficient

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Abstract: In this study energy dissipation coefficient and entropy generation process in characteristic wall shear flows have been investigated. Effect of pressure gradients on energy dissipation coefficient for flows undergoing “bypass” transition from laminar to turbulent state has been studied. Reynolds-Averaged Navier-Stokes (RANS) models and Direct Numerical Simulations (DNS) are implemented to study the energy dissipation coefficient and local entropy generation in pre-transitional and transitional regions. Three of these RANS models are transitional models such as, traditional SST $k-\omega$ (2eq), SST $k-\omega$ (4eq) and $k-kl-\omega$ and the results are compared with DNS. Four simulations have been performed for (1) zero, (2) favorable, (3) adverse and (4) strong-adverse pressure gradient cases. The numerical results show that the pressure gradient has a significant effect on energy dissipation coefficient and entropy generation.

Keywords: Energy dissipation coefficient, entropy generation, pressure gradients, transitional boundary layer, bypass transition, rans Models.

1. INTRODUCTION

Fundamental understanding of local (pointwise) distributions of entropy generation rates and energy dissipation coefficient in characteristic wall shear flows are significant to potentially increasing energy efficiency and sustainability and thereby reducing fuel consumption [1]. For entropy generated by fluid friction, the rates are predictable for developed turbulent flows and pure laminar flows as reported by McEligot *et al.* [2-4]. The main difficulty lies in prediction of flows undergoing a so-called “bypass” transition [5-7] from a laminar to a turbulent state induced by strong freestream turbulence and streamwise pressure gradients. Most of the work to date have been on investigating the flow field using direct numerical simulation and very few studied have been done on investigating the entropy generation and energy dissipation coefficient in such flow fields; fewer studies considered turbulence models like RANS [5-7] to examine the capability and versatility of such industrial turbulence models since the applications of DNS are confined to the flow field with low Reynolds number. Almost all of these works have not considered the effect of pressure gradients which have an important influence on such type of flows. Thus a study on entropy generation and energy dissipation coefficient employing different turbulence models were deemed of interest. E. Ghasemi *et al.* [5] studied the entropy

generation in a transitional boundary layer region under the influence of freestream turbulence numerically for zero pressure gradient case. As their results [5] show, all the RANS models which they studied predicted transition onset prematurely and, consequently, over predict the integral entropy generation rate and the skin friction coefficient in the transition region; and also their results show that among the RANS models, SST $k-\omega$ (4eq) model and $k-kl-\omega$ provide the better results and SST $k-\omega$ (4eq) model predicts the transition onset closer to DNS. E. Ghasemi *et al.* [5] results clearly show that how entropy generation evolves in the transition from laminar to turbulent and the rate of entropy generation in transition region is much greater than that value in the other regions (laminar and turbulent). E. Ghasemi *et al.* [5] investigated an approximate equation for pointwise entropy generation and energy dissipation coefficient; their results provide a very good estimate of evaluating the rate of entropy and energy dissipation coefficient which significantly reduces the computational cost for numerical methods to calculate the fluctuation velocities in the region very close to wall. In continuation of their work E. Ghasemi *et al.* [6,7] studied the effects of adverse and favorable pressure gradients on entropy generation. Their results show that transition is early and abrupt for the strong adverse pressure gradient but occurs further downstream and is longer with the increasingly favorable flows, and that entropy generation rate at the wall increases as pressure gradient grows.

The objectives of the present research is to perform a study on effects of pressure gradients on energy dissipation coefficient and entropy generation in a

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transitional boundary layer region over a flat plate under the influence of free stream turbulence.. Study of pressure gradient effects on entropy generation and energy dissipation coefficient have been performed for different cases of favorable, zero, adverse and strong-adverse pressure gradients. Numerical results of the turbulence RANS models have been compared to the DNS data to examine the accuracy of these models in modeling such flow fields which have a variety of industrial and engineering applications.

2. ENERGY DISSIPATION COEFFICIENT (C_d)

The entropy generation rate per unit area S'' is a key quantity for minimizing thermodynamic losses for overall design. In wall coordinates, it can be written as [5-7],

$$(S'')^+ = \frac{TS''}{\rho u_\tau^3} = \int_0^{y^+} (S'''' \{y^+\})^+ dy^+ \quad (1)$$

If the pointwise mean turbulence kinetic energy equation is integrated from the wall to the free stream - subject to the presence of free stream turbulence at the edge of the boundary layer - and the direct dissipation is added, the following can be derived as [8,9],

$$\begin{aligned} (S'' \{ \delta \})^+ &\approx \int_0^\delta (\partial U^+ / \partial y^+)^2 dy^+ - \int_0^\delta (\overline{uv})^+ (\partial U^+ / \partial y^+) \\ &dy^+ - \int_0^\delta \left[(\overline{u^2})^+ - (\overline{v^2})^+ \right] (\partial U^+ / \partial x^+) dy^+ - (d/dx^+) \\ &\int_0^\delta U^+ (1/2) (\overline{q^2})^+ dy^+ - (1/2) v_\delta^+ \left[(\overline{u_\delta^2})^+ + (\overline{v_\delta^2})^+ + (\overline{w_\delta^2})^+ \right] - \overline{v_\delta^+ p_\delta^+} \end{aligned} \quad (2)$$

In terms of freestream velocity, U_∞ , the entropy generation rate per unit area $(S'')^+$ may be written as an energy dissipation coefficient [8,9]

$$C_d = \frac{TS''}{\rho U_\infty^3} = (S'' \{ \delta \})^+ (C_f / 2)^{3/2} \quad (3)$$

3. NUMERICAL SIMULATIONS AND BOUNDARY CONDITIONS

The results of direct numerical simulation by Nolan and Zaki [10] are employed as benchmarks to assess the possible use of popular turbulence and transition models to predict entropy generation in bypass transition. The flow domain and boundary conditions are the same as those in E. Ghasemi *et al.* [5-7]. The grid spacing is uniform in the streamwise and clustered inside the boundary layer in the wall-normal direction. ASYSY-FLUENT is used for applying RANS and transitional models. The Finite-Volume (FV) approach

using the SIMPLE algorithm is used with second-order upwind differencing for the convective terms and the central differencing for diffusion terms.

4. RESULTS AND DISCUSSIONS

4.1. Zero Pressure Gradient

Figure 1 demonstrates the DNS prediction of the different terms contributing to the energy dissipation rate in Eq. (2). At the beginning of the computational domain, at $Re_{\delta_0} = 800$ where the velocity profile is laminar with very small fluctuations, the Blasius and DNS solutions nearly agree since the turbulent portion is still essentially zero.

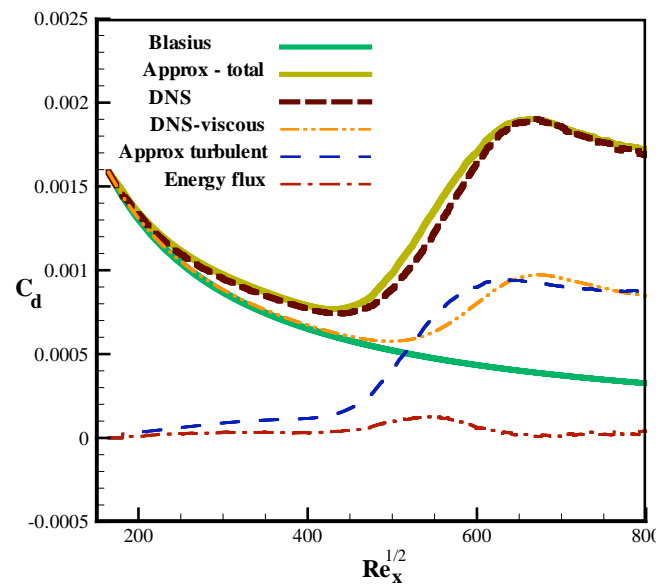


Figure 1: DNS results for dissipation coefficient C_d and related contributions.

As the flow develops the mean velocity profile deviates from the Blasius solution due to variation of the viscous term contribution. The quasi-turbulent contribution to C_d is enhanced in the pre-transitional region as the freestream turbulence initiates streaks growing inside the boundary layer. As seen from the Figure the contribution of the turbulent fluctuation is slightly greater than the viscous one in part of the transition region. There is a difference between 'approx-total DNS' and the exact DNS within the transition region that shows the contribution of the energy flux term in the energy dissipation equation. This energy flux term is first noticeable in the pre-transitional laminar boundary layer. It then grows to about ten per cent of C_d . As the developed turbulent boundary layer is approached it decreases and becomes negligible. This behavior is consistent with the observations of Walsh *et al.* [11]; their higher

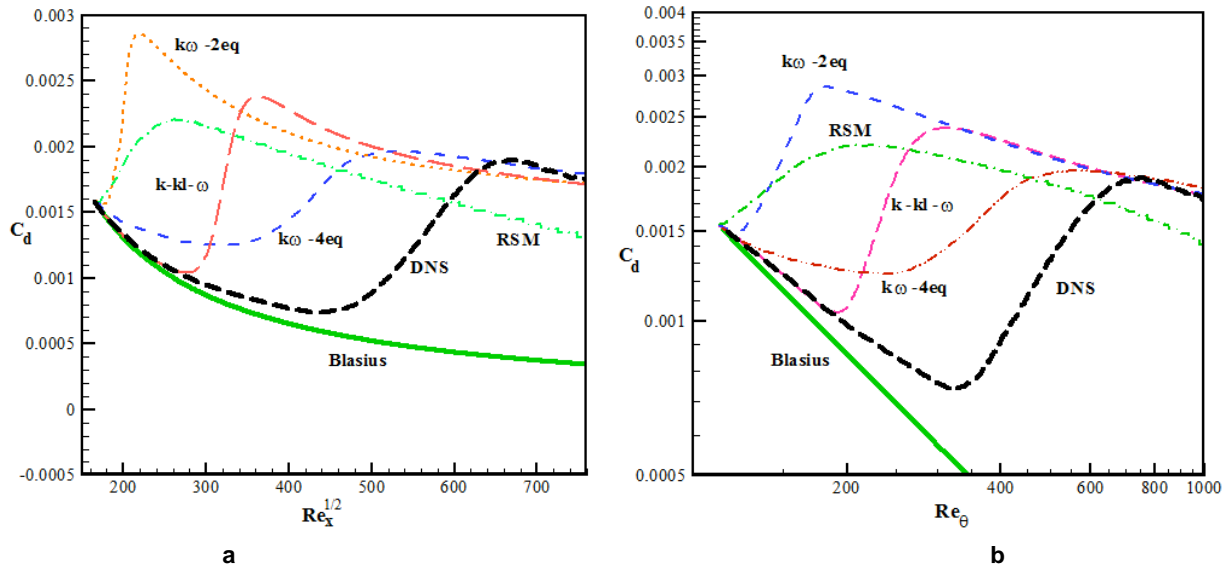


Figure 2: Comparison of dissipation coefficients C_d calculated by RANS models with total C_d by DNS.

freestream turbulence ($Tu_{in} = 4.7\%$) apparently led to a somewhat larger contribution from this energy flux term. The present RANS predictions of C_d are compared to the DNS results and Blasius predictions in Figure 2 in two formats for the reader's convenience.

As with C_f , predictions for the two turbulence models diverged from the pure laminar case almost immediately. The two transition models diverged more gradually, consistent with their later apparent transition onset, but still much sooner than the DNS. Consequently, all these RANS models overpredict the entropy generation rate in the transition region. For Re_θ greater than about 700, all except the RSM agree with the DNS results for the developed turbulent boundary layer [5-7, 12].

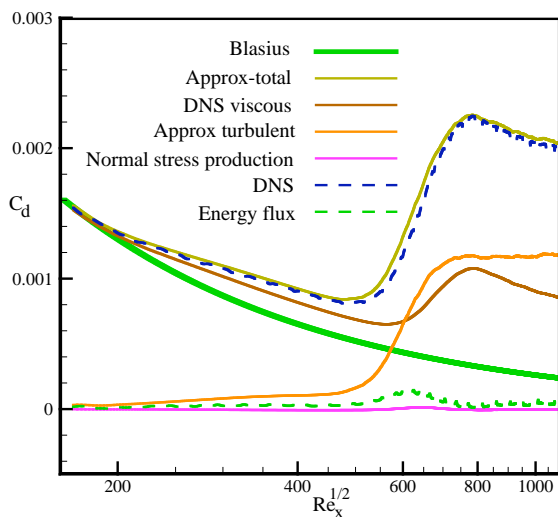


Figure 3: DNS results for dissipation coefficient C_d and related contributions.

The results of the present study can be extended by using LES modeling of turbulence flow [13-15], analytical modeling of boundary layer flow [16-23] or free convection heat transfer [24-29] using various numerical methods [30-33].

5. RESULTS FOR THE EFFECTS OF PRESSURE GRADIENTS

5.1. Favorable Pressure Gradient

Figure 3 demonstrates the DNS prediction of the different terms contributing to the energy dissipation rate in Eq. (2) for the FPG case. At the beginning of the computational domain, at $Re_{\delta_0} = 800$ where the velocity profile is laminar with very small fluctuations, the Blasius and DNS solutions nearly agree since the turbulent portion is still essentially zero. As the flow develops the mean velocity profile deviates from the Blasius solution due to variation of the viscous term contribution. The quasi-turbulent contribution to C_d is enhanced in the pre-transitional region as the freestream turbulence initiates streaks growing inside the boundary layer. As seen from the Figure the contribution of the turbulent fluctuation is slightly greater than the viscous one in part of the transition region. As can be seen from the Figure, the values of different terms are slightly greater than those values for the ZPG case which is due to the effects of favorable pressure gradient or decreasing the pressure gradient effects. There is a difference between 'approx-total DNS' and the exact DNS within the transition region that shows the contribution of the energy flux term in the energy dissipation equation. This energy flux term

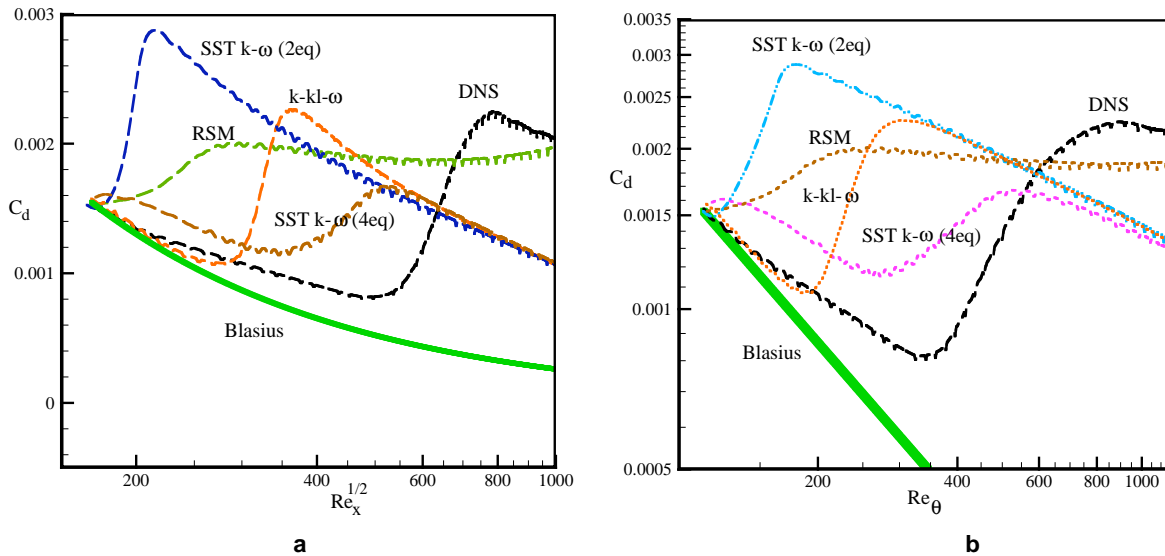


Figure 4: Comparison of dissipation coefficients C_d calculated by RANS models with total C_d by DNS.

is first noticeable in the pre-transitional laminar boundary layer. It then grows to about 10% of C_d . As the developed turbulent boundary layer is approached it decreases and becomes negligible. The present RANS predictions of C_d are compared to the DNS results and Blasius predictions in Figure 4 for the FPG case. As with C_f , predictions for the two turbulence models diverged from the pure laminar case almost immediately. The two transition models diverged more gradually, consistent with their delayed transition onset, but still much earlier than the DNS. Again compared to the ZPG case the DNS values of the FPG case are higher than the values for the ZPG case. Consequently, all these RANS models overpredict the entropy generation rate in the transition region. For

Re_θ greater than about 800, none of them except the RSM agree with the DNS results for the developed turbulent boundary layer.

5.2. Adverse Pressure Gradient and Strong Adverse Pressure Gradient

Figure 5 demonstrates the DNS prediction of the different terms contributing to the energy dissipation rate for the APG cases. At the beginning of the computational domain, at $Re_{\delta_0} = 800$ where the velocity profile is laminar from a ZPG (Blasius) with very small fluctuations, the Blasius and DNS solutions nearly agree since the turbulent portion is still essentially zero. As the flow develops the mean velocity profile deviates from the Blasius solution due to

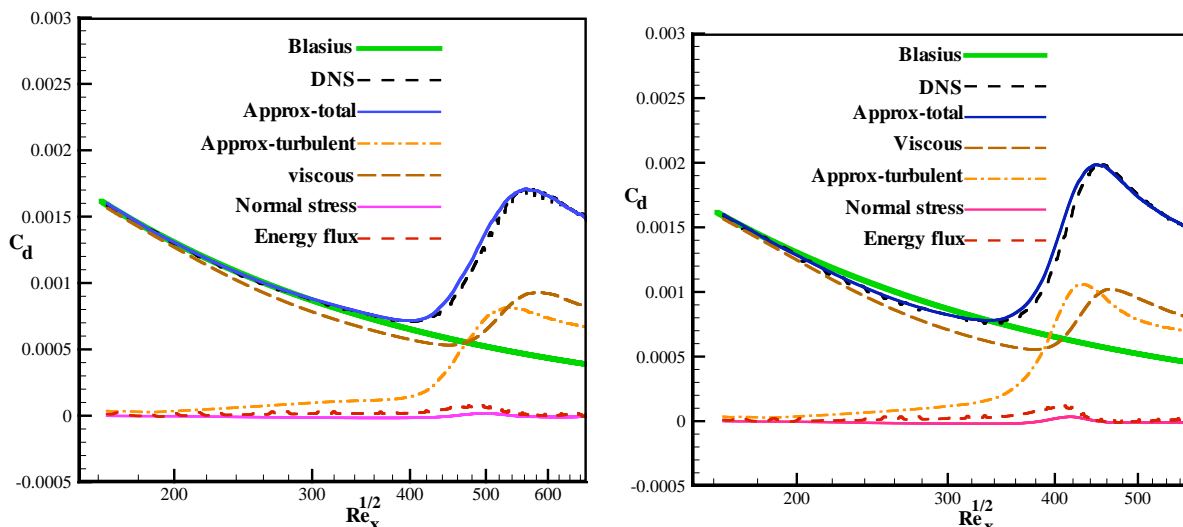


Figure 5: DNS results for dissipation coefficient C_d and related contributions, Top: APG , Bottom: APG_{strong} .

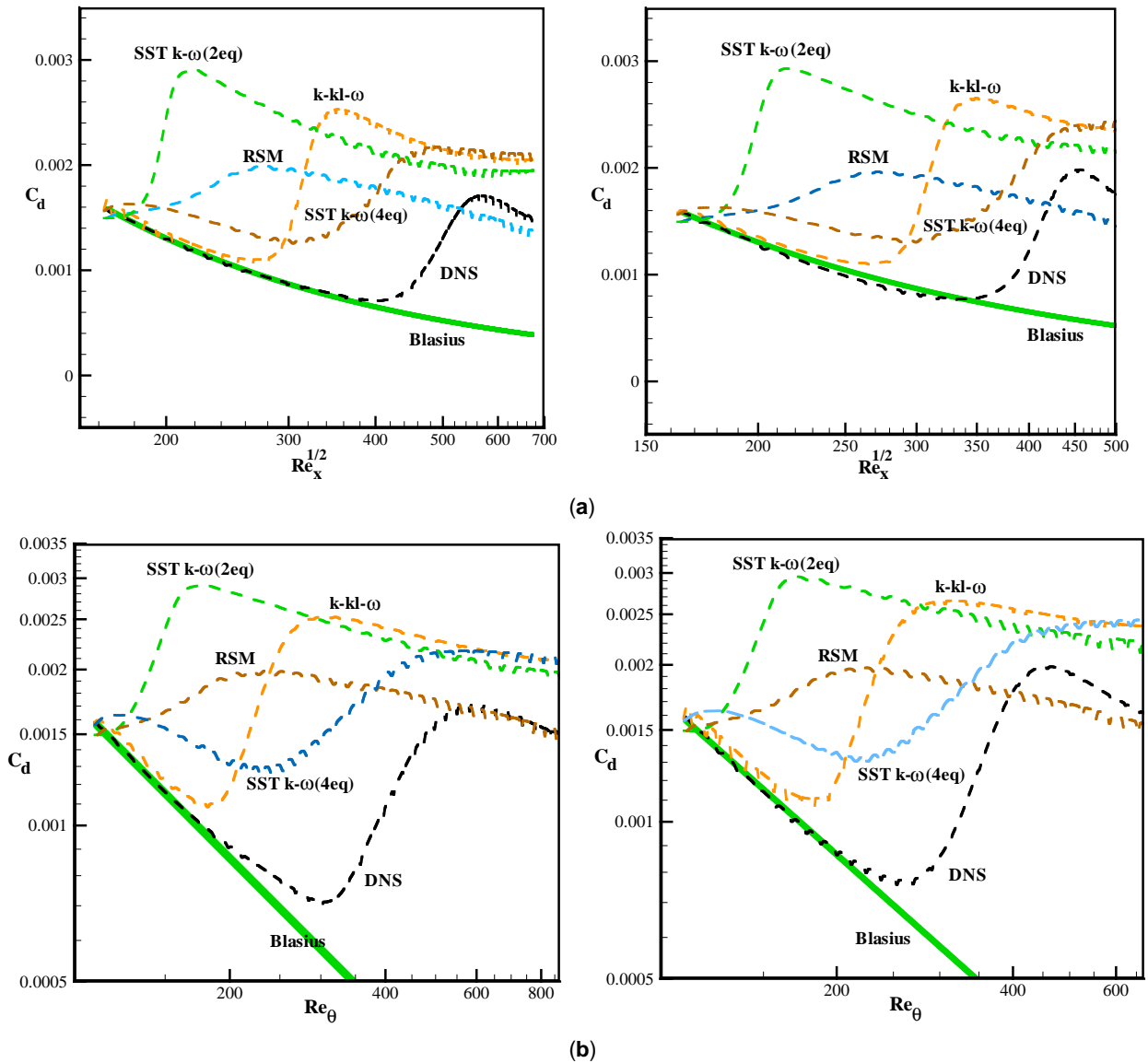


Figure 6: **a:** Comparison of dissipation coefficients C_d calculated by RANS models with total C_d by DNS. Top: APG, Bottom: APG_{strong} . **b:** Comparison of dissipation coefficients C_d calculated by RANS models with total C_d by DNS. Top: APG, Bottom: APG_{strong} .

variation of the viscous term contribution and pressure gradient. The quasi-turbulent contribution to C_d is enhanced in the pre-transitional region as the freestream turbulence initiates streaks growing inside the boundary layer. As can be seen from the Figure, the values of different terms are slightly lower than those values for the other cases which are due to the effects of adverse pressure gradients or increasing the pressure gradient effects. The present RANS predictions of C_d are compared to the DNS results and Blasius predictions in Figure 6 for the APG cases. As with C_f , predictions for the two turbulence models diverged from the Blasius case almost immediately. The two transition models diverged more gradually,

consistent with their delayed transition onset, but still much sooner than the DNS. Compared to the ZPG and FPG cases, the DNS values of the APG case is smaller than that value for the ZPG case, but for APG_{strong} it is almost the same as ZPG case.

6. CONCLUSIONS

Here, five RANS models ($k-\epsilon$ enhanced wall, SST $k-\omega$ (2eq), SST $k-\omega$ (4eq), $k-kl-\omega$ and RSM) have been implemented to study the energy dissipation coefficient and entropy generation in a transitional boundary layer region for flow over a flat plate, under the influence of free stream turbulence.

Study of pressure gradient effects on energy dissipation coefficient and entropy generation have been investigated for different cases of favorable, zero, adverse, and strong-adverse pressure gradients (FPG, ZPG, APG and strong APG). Numerical results of the turbulence RANS model based simulation have been compared to DNS data to examine the accuracy of these models in modeling such flow fields. All the RANS models over predict the energy dissipation coefficient and the integral entropy generation rate in the transition region.

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