A Parametric Study of Heat Transfer for the Optimization of Fin Sinks

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Abstract: Fin heat sinks are the most widely used type of heat sink for cooling purposes nowadays where space is a key factor, such as for the cooling of electronic equipment. Improved cooling capacity and the lowest possible thermal resistance in the design optimization process of these sink geometries means that we should consider a number of variable parameters, which can involve tedious design processes that are almost impossible to approximate to a sufficient degree of accuracy without computer simulation tools. The principal parameters are the heat dissipation base area, fin size, shape and material and the heat transfer coefficient. Computer numerical simulation tools greatly assist the design process, allowing in turn a greater range and more accurate analysis of the problem itself. In this study, we develop a design tool called "Opti-fin" for a Matlab ® environment that allows the user to configure a fin on the basis of the material and the thermal heat that will be released. Our study also includes a realistic estimation of fluid (air) flows that control the temperature dependency of the fin. This tool has been validated by computational fluid dynamic simulations using ANSYS-FLUENT®, in which the results of the simulation and the actual triangular shaped fin showed a remarkable similarity.

Keywords: Efficiency, fins, heat transfer, conduction and convection, parametric study, Matlab, CFD.

1. INTRODUCTION

Fin heat sinks [1, 2] are additional areas installed at certain locations to increase the interface between the heat exchange surface and the surrounding environment in a given piece of equipment [3]. They are nowadays the most frequently used cooling strategy where space is a key factor, as in the case of electronic equipment [4], or when the coefficients of convective heat transfer between the solid and the medium present low values, as occurs in some cases of natural convection [5, 6, 7]. Thus, low coefficients can in some way be compensated, by increasing the heat transfer area in contact with the fluid, as the heat power that is delivered is given by:

$$\dot{Q} = \alpha \cdot S \cdot \left(\theta_s - \theta_a\right) \tag{1}$$

The design of this type of heat sink implies the lowest thermal resistance for a given size, following a tedious design process that involves a series of parameters that are almost impossible to approximate to an acceptable degree without the use of computa-tional tools. These tools are of great assis-tance in this design process [8], because of the wider range and the accuracy of the calculations that are required to solve the problem.

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Fin configurations currently on the market present a myriad of sophisticated geometries [9, 10]. These settings are designed for very specific applications that require high performance under maximum constraints in relation to space [11] and materials [12]. However, the bulk of the applications employing additional surfaces to enhance simple heat transfer geometries use longitudinal fins (among which, "rectangular" root straight surfaces with constant profiles and "triangular" fins with reduced sections along the generatrix) and cylindrical "ring" surfaces with a constant starting and generating profile [13]. There is also the special case of either conical or cylindrical fins called "needles". We have developed a thermal design tool from these configurations based on Matlab, which we refer to as "Opti-fin" [14].

In this paper, we compare the performance of the "Opti-fin" tool for a triangular fin with the results of CFD modelling, validated with the ANSYS-FLUENT software package. A remarkable similarity in the results of both models may be observed. The thermal behaviour of a triangular fin along its length and width are presented, as well as a comparison between a triangular fin and a rectangular fin of equal length.

2. AIMS AND METHODOLOGY

The aim of this study is to present a new tool in the Matlab environment that calculates the heat transference of fins, contributing to quick and reliable

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design techniques. The “Opti-fin” tool is able to describe the thermal behaviour of rectangular, triangular and annular fins, although the triangular profile of fins are examined in the present work, so that the influence of design parameter variations on their effectiveness and in terms of thermal power and dissipation may be analyzed.

We used an analytical formulation of the heat transfer phenomena for the development of this tool [15, 16]. The basic assumptions of this formulation should be borne in mind by the user when assessing its usefulness.

3. NUMERICAL MODEL FORMULATION

The “Opti-Fin” tool is capable of estimating the temperature field, power dissipation, and the efficiency of a triangular fin, the basic parameters [17] of which are shown below in Figure **1**.

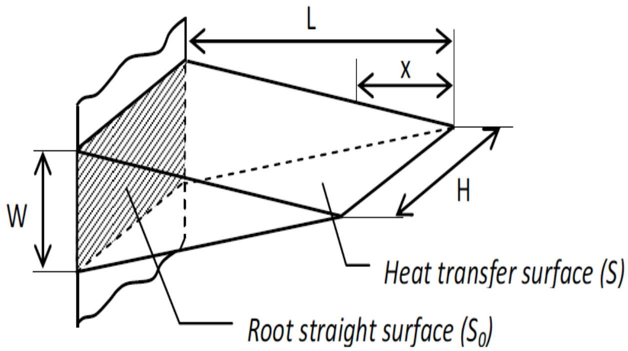

Figure 1: Geometric parameters of the triangular fin.

The basic geometric parameters of this fin are the width of its base “W”, height “L” and depth “H”. The following assumptions apply in this study:

- Forced air convection across the periphery and the tip of the fin.
- The fin material is aluminium.
- The temperature at the base of the fin is fixed.
- Fin depth is constant.
- The fin is thin enough to consider unidirectional flow.
- Heat losses from the sides of the fin are ignored.

As shown in the figure below, the exchange surface of the fin, is a function of “x”,

$$S_x = 2 \cdot H \cdot x \cdot \sqrt{1 + \left(\frac{W}{2 \cdot L}\right)^2} = 2 \cdot H \cdot x \cdot f \tag{2}$$

where, f is the geometric factor of the triangular fin. Therefore, if x = L, the overall upper and lower (S) exchange surface and the surface power (S_0) may be respectively defined as:

$$S = 2 \cdot H \cdot L \cdot f \tag{3}$$

$$S_o = W \cdot H \tag{4}$$

3.1. Temperature Field

The Biot number is defined as:

$$Bi = \frac{\alpha \cdot L_c}{\lambda} \tag{5}$$

Where L_c is the characteristic length of the fin, which is commonly defined as the volume divided by the surface area of the body.In our case, the value of the Biot number, smaller than 0.1 implies that the heat conduction inside the body is much faster than the heat convection away from its surface, so temperature gradients are negligible inside of it. According to this, the temperature field along a triangular body (fin), is defined by the expression [18]:

$$\frac{d^2\beta}{dX^2} + \frac{1}{X} \cdot \frac{d\beta}{dX} - \frac{\varphi^2}{4} \cdot \frac{\beta}{X} = 0 \tag{6}$$

Where:

$$\beta = \frac{\theta - \theta_a}{\theta_0 - \theta_a} \tag{7}$$

$$\varphi = \sqrt{\frac{8 \cdot \alpha \cdot f \cdot L^2}{\lambda \cdot W}} \tag{8}$$

$$X = \frac{x}{L} \tag{9}$$

Solving the second-order differential equation and applying the appropriate boundary conditions, the following expression for the temperature field is obtained:

$$\beta = \frac{I_0\left(\varphi \cdot \sqrt{X}\right)}{I_0(\varphi)} \tag{10}$$

Where I_0 is the modified Bessel function of the first kind of order zero.

3.2. Thermal Power Dissipated

The thermal power dissipated by the triangular fin is given by the following expression:

$$\dot{Q} = -\lambda \cdot S_0 \cdot \left(\frac{d\beta}{dX}\right)_{X=L} = \ldots = -\frac{2 \cdot f \cdot a \cdot L \cdot \alpha}{\varphi} \cdot 2 \cdot \frac{I_1(\varphi)}{I_0(\varphi)} \cdot (\theta_0 - \theta_a) \quad (11)$$

Where I_1 is the modified Bessel function of the first kind of order one.

3.3. Efficiency

Finally, the yield of a triangular shaped fin is given by the expression:

$$\eta = \frac{2}{\varphi} \cdot \frac{I_1(\varphi)}{I_0(\varphi)} \quad (12)$$

4. VALIDATION OF THE TOOL THROUGH CFD

4.1. Building the CFD Model

The results of the tool developed in Matlab were compared with those from CFD simulation using the ANSYS-Fluent [19] software package for validation purposes. An aluminium fin with a triangular profile, a base length of L = 50 mm and a width of W = 3 and 10 mm was chosen, as an example. The boundary conditions for air and body of the fin are:

- Ambient temperature θ_a = *25 °C*.
- Temperature at the base of the fin θ_0 = *80 °C*.
- Outer convection coefficient α = *10 W/m² K*.
- Thermal conductivity of aluminium λ = *146 W/m K*.

4.2. Sensitivity Analysis

A sensitivity analysis for the mesh was first performed, including the study of 8 meshes with a different design to ensure grid independence. The mesh sizes are shown in Table **1**. The starting mesh has a minimum element size of 0.05 mm and a maximum size of 1 mm. The final mesh has a minimum element size of 0.005mm and a maximum size of 0.01mm.

In all cases, the variation of the average temperature of the fin was monitored. Element size decreased until no change was observed in the temperature. In Table **1**, design point DP 1 represents the starting coarse mesh and design point DP 8 shows the optimized mesh.

Table 1: Sensitivity Analysis for the Mesh

Design point	Min Size (mm)	Max Size (mm)
DP 1	0.05	1
DP 2	0.04	0.8
DP 3	0.03	0.6
DP 4	0.02	0.4
DP 5	0.01	0.2
DP 6	0.009	0.18
DP 7	0.008	0.16
DP 8	0.005	0.01

Figure **2** shows a detail of the cells for two of the different meshing schemes. The number of cells in the coarser mesh (Figure **2a**) was about 373, while in the optimized mesh (Figure **2b**) it reached 31,677 cells.

The CFD analysis was performed in 2D, in double precision with a third-order discretization for the energy equation.

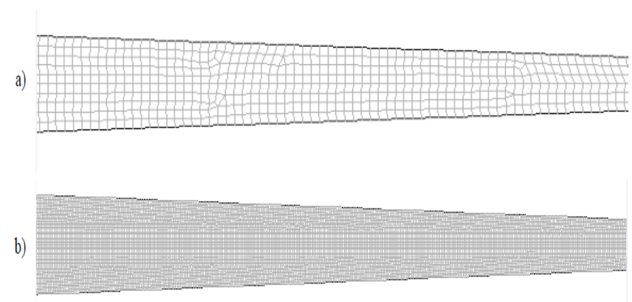

Figure 2: Detail of the cells in the CFD models, **a**) Starting mesh, **b**) Final mesh.

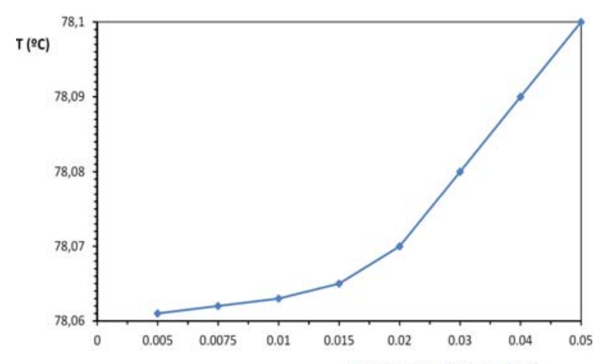

Figure 3: Evolution of the average temperature of the fin by size of element.

Figure **3** shows the evolution of the average temperature on the fin by the size of each element. Little variation in the average temperature of the fin may be observed for values lower than the minimum size of 0.02mm.

It was decided to use a minimum element mesh size of 0.01mm for comparison and validation of the results obtained with the “Opti-fin” tool.

4.3. Set-up of the Experiments

Under these conditions and using the ANSYS - Fluent CFD software, the variations in the surface temperature of the fin were analyzed for two different fin widths of W = 3mm and 10mm. respectively along their length. The geometric details of the fins are shown in Figure **4**.

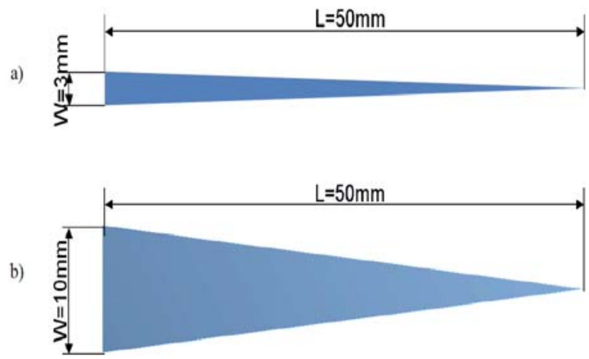

Figure 4: Fin geometries studied, **a**) W = 3mm and **b**) W = 10mm.

The temperature field provided by the CFD model for this configuration is outlined in Figure **5** for fin widths of 3 and 10 mm. respectively.

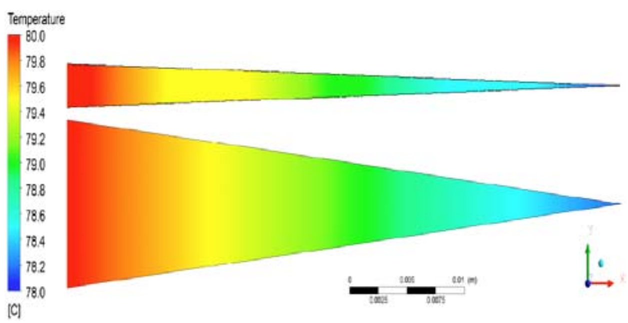

Figure 5: Detail of the CFD surface fin temperatures for W = 3 and 10 mm. respectively.

While the final comparative temperature outputs through *Fluent* and *Opti-fin* for W = 3 and 10 mm. respectively are depicted in Figure **6**.

It is evident from the above Figures that the "Opti-fin" tool accurately predicts the temperature distribution on the surface. The temperature prediction error at the tip of the fin decreases as the W / L ratio increases. In the case of W = 3mm (W/L = 0.06), the difference is 2 °C while the difference is reduced to 0.4 ºC in the case of W = 10mm (W/L = 0.2). The maximum deviation stands at around 3%, which validates the "Opti-fin" tool.

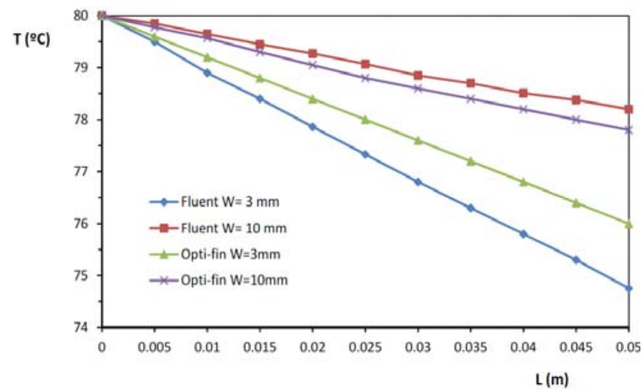

Figure 6: Temperature distributions in fluent and *Opti-fin* for W = 3 and 10 mm. respectively.

After checking the validity of the tool, the effect of base width and length on heat transfer was studied in the context of the triangular fin.

5. RESULTS

The results obtained for a triangular fin under different geometrical configurations are shown, so as to assess the effect of its capacity on heat dissipation.

5.1. Effect of Variations in Width “W”

Both the length L = 50 mm and the depth H = 450 mm of the fin are considered fixed for this analysis, so the independent variable is the width W. In Figure **7**, the efficiency of the fin is shown from the base to the tip under thermal conditions of 80 °C at the base, at an ambient temperature of 25 °C.

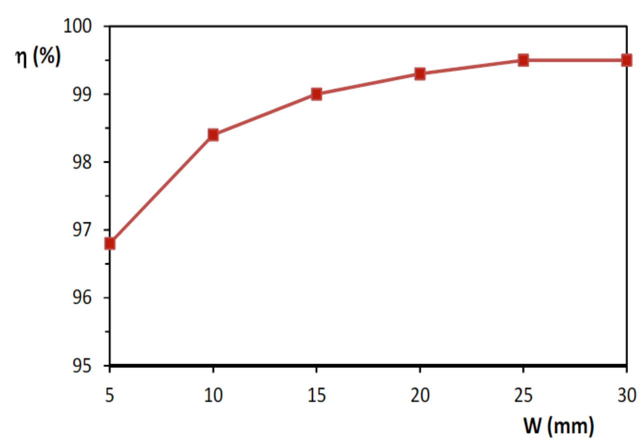

Figure 7: Effect of fin width on efficiency.

It may be observed that a wider fin width, leads to greater the efficiency [20, 21]. However from a certain width (15 mm in this case), the increase in efficiency is negligible. Therefore, W = 15mm may be considered the width limit.

5.2. Effect of Variations in Length "L"

In this case, three lengths were analyzed: 50, 70 and 100 mm for a fin width of W = 5mm and a depth of H = 450 mm. The results are shown in Figure **8**, where we can see the surface temperature for the above-mentioned lengths.

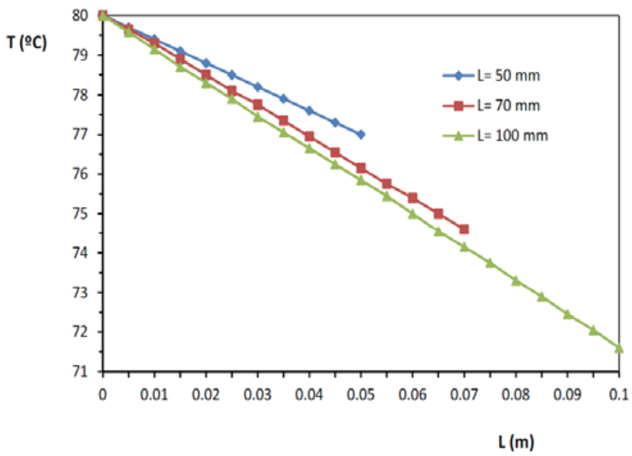

Figure 8: Surface temperature of the fin for L = 50, 70 and 100 mm. respectively.

As the fin length increases, the temperature at its tip is reduced, so that the dissipated power is also slightly reduced. Increased length is therefore related to decreased efficiency [22], as shown in Table **2**.

Table 2: Effect of Fin Length on Dissipated Power

L (mm)	$\dot{Q}$ (W)
50	95.13
70	95.05
100	95

5.3. Comparative Rectangular VS. Triangular Fin

A design decision between either a rectangular or a triangular fin often arises when designing a heat sink. In this section, the relative efficiencies of both configurations are compared for a length L = 50 mm with the same outer surface "S" of the heat exchange. Figure **9** shows how the temperature at the tip of the rectangular fin is lower than that of the triangular fin. These results point to the increased thermal resistance of the rectangular shape as opposed to the triangular shape with the same exchange surface.

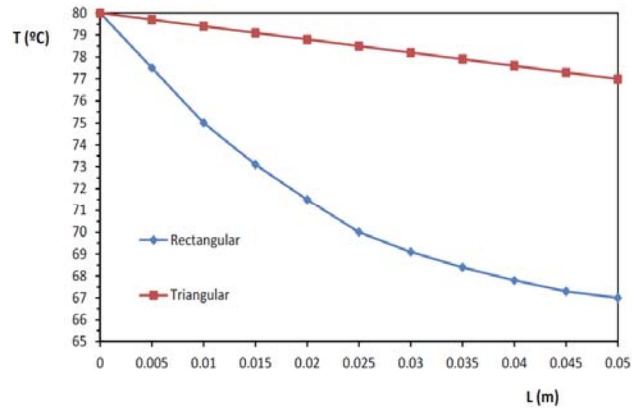

Figure 9: Surface temperature distribution for rectangular and triangular fin respectively.

The efficiency of the triangular fin is 1.04 times greater than that of the rectangular fin:

$$\frac{\eta_T}{\eta_R} = 1.04 \tag{13}$$

CONCLUSIONS

The following conclusions can be outlined from this study:

The "Opti-Fin" tool under predicts temperature values by a maximum of 3% with regard to the results obtained by CFD with ANSYS FLUENT; a result that, in our opinion, validates its use as a design tool.

As has been shown, it is a simple and quick tool for the optimal design of fins with effective results.

Comparative studies have been conducted to evaluate the influence of the different parameters of the fin (such as width and length) on the performance of the fin in terms of cooling capacity.

Furthermore, the effect of fin shape (triangular versus rectangular) on efficiency was compared, with better results for the triangular rather than the rectangular shape with the same heat-exchange outer surface.

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NOMENCLATURE

CFD = Computational Fluid Dynamics

f = Geometrical factor of the fin

H = Depth of the fin (mm)

I_0 = Modified Bessel function of the first kind of order zero

I_1 = Modified Bessel function of the first kind of order one

L = Length of the fin (mm)

L_c = Characteristic length of the fin

$\dot{Q}$ = Thermal power released by the fin (W)

S = Heat exchange surface of the fin (mm^2)

S_0 = Root surface of the fin (mm^2)

W = Width of the fin (mm)

Greek Symbols

α = External convection coefficient (W/m^2 K)

β = Relative temperature

θ_a = Ambient temperature (ºC)

θ_0 = Temperature at the root of the fin (ºC)

θ_s = Temperature at the surface of the fin (ºC)

λ = Thermal conductivity (W/m K)

η_T = Efficiency of the triangular fin

η_R = Efficiency of the rectangular fin

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